HYDRODYNAMIC NOTE AG-25

DERIVATION OF AND INTRODUCTION TO FLAP LIFT CAVITATION EQUATIONS AND CAVITATION BUCKETS

by

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>2</td>
</tr>
<tr>
<td>Conclusions</td>
<td>4</td>
</tr>
<tr>
<td>Discussion</td>
<td>5</td>
</tr>
<tr>
<td>PART I: Basic Theory and Derivation of Equations</td>
<td></td>
</tr>
<tr>
<td>PART II: Reference Life Coefficients</td>
<td>20</td>
</tr>
<tr>
<td>PART III: Construction of the Cavitation Buckets</td>
<td>24</td>
</tr>
<tr>
<td>PART IV: Optimization Considerations</td>
<td>33</td>
</tr>
<tr>
<td>APPENDIX I: Sources of Error in $C_{LO}$</td>
<td>35</td>
</tr>
<tr>
<td>APPENDIX II: Alternate Method of Foil Cavitation Bucket Construction</td>
<td>38</td>
</tr>
<tr>
<td>APPENDIX III: Verification of Foil Cavitation Bucket Program</td>
<td>41</td>
</tr>
<tr>
<td>APPENDIX IV: Definition of $C_{\text{eff}}$</td>
<td>43</td>
</tr>
<tr>
<td>References</td>
<td>46</td>
</tr>
<tr>
<td>Symbols</td>
<td>47</td>
</tr>
<tr>
<td>Table I: Cavitation Parameters, AG(EH) Forward Foil Calculation</td>
<td>49</td>
</tr>
<tr>
<td>Table II</td>
<td>50</td>
</tr>
<tr>
<td>Table III: Results of Foil Cavitation Bucket Equation Calculation</td>
<td>51</td>
</tr>
<tr>
<td>Table IV</td>
<td>52</td>
</tr>
<tr>
<td>Table V</td>
<td>53</td>
</tr>
<tr>
<td>Table VI</td>
<td>54</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1: Illustrative Example of Section Cavitation Bucket 9
Figure 2: Definition of q' 11
Figure 3: Illustrative Example of Foil Cavitation Bucket 15
Figure 4: Foil Cavitation Bucket, AG(EH) Forward Foil; Incidence Lift 19
Figure 5: Flap Basic Load Distribution 25
Figure 6: Section Cavitation Bucket, AG(EH) Forward Foil, Flap Lift 26
Figure 7: Foil Cavitation Bucket, AG(EH) Forward Foil, Flap Lift 28
Figure 8: Relationship Between Flap Lift, Incidence Lift and W/S ref Curves 31
Figure 9: Flap Cavitation Characteristics 32
SUMMARY:

The object of this note is to provide an introduction to a comprehensive and detailed study of flap lift which is to follow this note.

A very powerful equation for flap lift cavitation has been derived as a result of all previous flap lift investigations. Up until this report flap control of cavitation was not a state of the art concept. Now that an easy to understand, accurate method of determining cavitation characteristics has been derived further applications and extensive investigations of flap control are possible at a greatly reduced cost since the need for costly computer time and extensive programs is no longer required.

Through the use of known equations for incidence lift and Allen's flap velocity distributions the foil cavitation equation was derived. A check was performed on the resulting bucket by assuming a zero pitch angle and trimmed flaps, and comparing it to the curve for zero flap deflection and the curve for incidence lift. Deflection angle equations for the desired lg case have been formulated through the use of "reference terms" which were in themselves derived as a necessity to this note.
INTRODUCTION:

The purpose of this note, first in a series of related notes on flap lift cavitation, is to establish the basis for a detailed and rigorous investigation of the flapped foil for use in hydrofoil designs.

This first note will include the following:

1. A review of previous flap lift cavitation work (Hydrodynamic Note AG-5), and incidence lift cavitation advancements (Hydrodynamic Note AG-18). This will establish the basic equations and concepts involved in the final flap lift cavitation equation.

2. The derivation of the basic section cavitation bucket for the flap lift case and a presentation of the results in graphical form with a practical example.

3. A derivation and illustration of the section cavitation bucket which evolves from the basic foil loading form w/s. Explanation of all new variables and symbols will be included.

4. A step by step discussion of the unique \(W/S\) \(_{\text{ref}}\) and \(W/S\)' terms.

5. Investigation of cavitation flap deflection angles, \(S_{\text{cavitation}}\).

6. Introduction to future flap lift hydrodynamic notes with a brief consideration of optimizing the characteristics of:
Employing the flap lift system to hydrofoil vehicles will expand the realm of hydrofoils in the future. First, larger foils with spans in excess of twenty feet will become possible without the excessively large control system required if only incidence lift control were employed. Second, the problem of cavitation will become slightly reduced by expanding the boundaries of the illustrative foil cavitation bucket.

All of the equations have been reduced, and all constants have been selected for application to the AG(EH) forward foil. Application to any other foil may of course be considered.
CONCLUSIONS:

Equations 7 and 14 of this note define the two dimensional section and the three dimensional foil cavitation-buckets for the flap lift case. The construction of these buckets allows visualization of how the flaps expand the region of cavitation-free operation, thus allowing performance at reduced speeds and greater foil loadings.

A breakdown of the foil cavitation bucket equation into those terms unique to flaps allows the construction of equations which determine allowable flap deflections for lift operation, and the first sign of cavitation. With the variation of pitch and incidence angles and the application of the flap deflection equations a complete catalogue of restricting flap angles can be compiled for stability and control purposes. The relationships formulated in this report shall provide a reference and a basis for all flap lift investigations.
DISCUSSION

In beginning this discussion and evaluation of flap lift and its cavitation characteristics two major assumptions must be made in order to proceed with the derivation of the cavitation bucket.

The first problem considered arises from the fact that the peak values for pitch lift, incidence lift, and flap lift do not occur at the same spanwise locations on the foil. So the first assumption made shall be that the largest \( (C_L/C_L)_{\text{max}} \) values that occur on each lift coefficient shall be the ones used in the derivation, disregarding their location on the foil. This will assure that the derived cavitation bucket will account for that critical station where the first sign of cavitation begins. Incorporating into the theory the concept of full span flaps the equality \( (C_L/C_L)_1 \times (C_L/C_L)_5 \) can be made for simplicity. So for a given foil configuration a total \( C_L \) can be determined, and since a normal to the quarter chord dynamic pressure \( q \) is known the total section foil loadings, \( w/s \), at the critical span station can be determined by the relation:

\[
C_L q = w/s
\]

This directs the theory to a second problem and assumption. If the \( C_L \)'s vary along the span, what is the average foil loading on the foil? This is necessary to determine because in investigating a single section at an arbitrary spanwise location a transformation to a foil cavitation bucket is not possible unless an average section foil loading can be
determined. No single $C_l/C_L$ ratio will carry the derivation from the section bucket to the foil bucket because the section loading has, as was stated before, three components associated with $C_l/C_L$ ratios. It is the decomposition of the section loading and the reassembly of the average foil loadings which constitute the bulk of the cavitation bucket. At this point a clarification in foil loading terminology should be made. WS from this point on will denote the foil loading for the three dimensional foil whereas $W/S$ will be the two dimensional section foil loading. The differing factor being $C_L$ and $C_l$ respectively.

The definition of $W/S$ is:

$$ (w/s)_0 + (w/s)_1 + (w/s)_0 + (w/s)_\alpha = W/S $$

This relationship can be derived from the terminology just discussed. The only unknown term for flap lift control is $(w/s)_0$:

$$ (w/s)_0 = W/S - (w/s)_1 - (w/s)_\alpha $$

The derivation of the section bucket will proceed from this point starting with Equation 37 of reference 1. The most basic local velocity distribution over the section can be written,

$$ \sqrt{S} = V/V \pm \Delta V/V \pm (C_l \mp C_{laf} - C_{leff}) \Delta a/V \pm C_l b (\Delta V/V)_f $$

where $b_l$ is basic flap loading and $C_{laf} = C_{lf} - b_l$, and where
\[ (4) \]
\[
\begin{align*}
C_{1b} &= \mathcal{S}(C_1 - C_1') \\
C_{1f} &= (1-\mathcal{S})(C_1 - C_1')
\end{align*}
\]

See Ref. 1 for further explanation

\[ C_1' = (C_1)' + C_{1\text{eff}} \]

So a general form of Equation 46 of Reference 1 can be written as,

\[ (5) \]
\[
\sqrt{S} = \frac{1}{\gamma_v} \pm \frac{a_{uv}}{\gamma_v} \pm \left[ C_{e'} + (1-\mathcal{S})(C_e - C_{e'}) - C_{e,\text{reff}} \right] \frac{a_{uv}}{\gamma_v}
\]
\[
\pm \mathcal{S} \left( \frac{a_{uv}}{\gamma_v} \right)_f (C_e - C')
\]
\[
= \frac{1}{\gamma_v} \pm \frac{a_{uv}}{\gamma_v} \pm \left[ (\mathcal{S} C_{e'} - C_{e,\text{reff}}) \frac{a_{uv}}{\gamma_v} \right] \pm \mathcal{S} \left( \frac{a_{uv}}{\gamma_v} \right)_f C_e
\]
\[
\pm \left\{ \frac{a_{uv}}{\gamma_v} + \mathcal{S} \left( (\mathcal{S} \frac{a_{uv}}{\gamma_v})_f - \frac{a_{uv}}{\gamma_v} \right) \right\} C_e
\]

This is now a very basic equation in the linear form of \( y = b + mx \).

It should be noted that the term \( C_1' \) includes both pitch and incidence lift since they are independent of the section. At this point \( C_1' \) will be used for camber and incidence lifts only. Equation 5 becomes,

\[ (6) \]
\[
\sqrt{S} = \frac{1}{\gamma_v} \pm \frac{a_{uv}}{\gamma_v} \pm \mathcal{S} \left[ C_{e'} + (C_v)_{\alpha'} \right] - C_{e,\text{reff}} \frac{a_{uv}}{\gamma_v}
\]
\[
\pm \mathcal{S} \left( \frac{a_{uv}}{\gamma_v} \right)_f \left[ C_{e'} + (C_v)_{\alpha'} \right] \pm \left\{ \frac{a_{uv}}{\gamma_v} + \mathcal{S} \left( (\mathcal{S} \frac{a_{uv}}{\gamma_v})_f - \frac{a_{uv}}{\gamma_v} \right) \right\} C_e
\]
The equation can be condensed by initiating the parameters $\gamma$ and $\omega$.

$$\sqrt{S} = \gamma \pm \omega \left[ C_l + (C_l)_{\infty} \right] \pm (\Delta p a/V - \omega) C_l$$

Note that this is still in the slope intercept form with $C_l = \gamma$.

Equation 7 is now the equation used to graphically represent the section cavitation bucket. Figure 1 shows a very general section cavitation bucket. It can be seen that the upper area is the allowable and most restricting operating area. A practical application of this representation is seen in the following example.

If a foil section were operating at point A, at a given $\sqrt{S}$, and at a given $C_l = L/2 \gamma^2 S$, where the S associated with the Cl is area, the section could increase its $C_l$ by decreasing its speed or increasing the lift. This would advance point A towards the right of the graph. Cavitation would not occur anywhere on the section until point A' was reached. At this location cavitation would occur on the upper leading surface. In examining point B as it moves towards the right of the graph it will incur cavitation on a middle chord station before cavitating on the leading stations. What this graph is basically showing is a linear connection of the low pressure regions of the foil section through a range of $C_l$'s or simply a range of speeds.

In order for the derivation to continue an explanation of "cavitation dynamic pressure" must be given.

Since the 16-0.39008 section for the AG(EH) is defined as a section normal to the quarter chord the velocity component that passes over the
FIGURE 1 Illustrative Example of Section Cavitation Bucket
normal to the quarter chord is the velocity responsible for producing the foil cavitation. See Figure 2. So a new definition of \( q \) must be given. Basically, through simple geometry,

\[ q' = q \cos^2 \Delta \]

Now reconsidering the slope intercept form of the cavitation bucket:

\[ \sqrt{S} \cdot \gamma = u [C_{\ell}^{'*}(C_{\ell})_a] \leq (u/v - u) C_{\ell} \]

Rearranging,

\[ \pm (u/v - u) C_{\ell} = \sqrt{S} - \gamma = u [C_{\ell}^{'*}(C_{\ell})_a] \]

\[ (u/v - u) C_{\ell} = \pm (\sqrt{S} - \gamma) - u [C_{\ell}^{'*}(C_{\ell})_a] \]

Multiplying through by \( q' \), since the interest is in the cavitation of the section and the section foil loading.

\[ (u/v - u)(w/s) = \pm (\sqrt{S} - \gamma) q' - u [C_{\ell}^{'*}(C_{\ell})_a] \]

For the flap lift case \( (w/s)' = (w/s)_1 + (w/s)_0 \).

\[ w/s = w/s)_1 (c_{\ell}')(c_{\ell})_1 + (w/s)_0 (c_{\ell})_1 + (w/s)_1 (c_{\ell})_0 \]

\[ = (w/s)_1 (c_{\ell})_1 - (w/s)_0 (c_{\ell})_1 + (w/s)_1 (c_{\ell})_0 - (w/s)_0 (c_{\ell})_0 \]
The component of velocity that is responsible for creating the cavitation on the foil is the normal component to the foil. Since there is an infinite amount of normals to a swept-tapered foil, the normal component to the quarter chord is used since the quarter chord marks the location of the aerodynamic center of the foil sections. Thus $V \cos \Delta$ is incorporated into the dynamic pressure equation to form $q'$. 

Figure 2 Definition of $q'$
Employing the parameter \( j \) into the equation,

\[
\left[ (\frac{\alpha}{\beta})_j - (\frac{\alpha}{\beta})_j^* \right] (\frac{\alpha}{\beta})_j
\]

\[
+ (\alpha_j^*) \left[ (\frac{\alpha}{\beta})_j - (\frac{\alpha}{\beta})_j^* \right] + \frac{\alpha_j}{\beta} \left[ (\frac{\alpha}{\beta})_j - (\frac{\alpha}{\beta})_j^* \right]
\]

\[
= \frac{\alpha_j}{\beta} \left[ (\frac{\alpha}{\beta})_j + (\frac{\alpha}{\beta})_j^* \right] (\frac{\alpha}{\beta})_j^* - 1 \left[ (\frac{\alpha}{\beta})_j - (\frac{\alpha}{\beta})_j^* \right]
\]

Employing the parameter \( j \) into the equation,

\[
\left( \frac{\alpha_j}{\beta} \right) = \frac{(\frac{\alpha}{\beta})_j - 1}{(\frac{\alpha}{\beta})_j^*}
\]

\[
\frac{\alpha_j}{\beta} = \frac{(\frac{\alpha}{\beta})_j - 1}{(\frac{\alpha}{\beta})_j^*}
\]

\[
\frac{\alpha_j}{\beta} = \frac{\alpha_j}{\beta} \left[ (\frac{\alpha}{\beta})_j - (\frac{\alpha}{\beta})_j^* \right] + \frac{\alpha_j^*}{\beta} \left[ (\frac{\alpha}{\beta})_j - (\frac{\alpha}{\beta})_j^* \right]
\]

Recalling

\[
\left( \frac{\alpha_j}{\beta} - \alpha_j \right) \frac{\alpha_j}{\beta} = \pm (\sqrt{5} - 2) \frac{1}{2} \left[ (\frac{\alpha_j}{\beta})^* + \frac{\alpha_j}{\beta} \right]
\]
and substituting the relations from Equation 9 and expressing the section loadings on the right in terms of foil loadings,

\[
\left( \frac{b}{v} - u \right) \left[ \frac{1}{5} \left( \frac{c}{x} \right)_6 + \frac{1}{5} \left( \frac{c}{x} \right)_i \right] \left( \frac{c}{x} \right)_6 \left( \frac{c}{x} \right)_i \\
\left( \frac{c}{x} \right)_i
\]

\[
\pm \left( \sqrt{5} - \psi \right) \left( \frac{v}{s} \right)' \left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \\
\left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_i \left( \frac{c}{x} \right)_v \\
\left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_i \left( \frac{c}{x} \right)_v \\
\left( \frac{c}{x} \right)_i
\]

\[
= \pm \left( \sqrt{5} - \psi \right) \left( \frac{v}{s} \right)' \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] - \left( \frac{v}{s} \right)_v \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] \\
= \pm \left( \sqrt{5} - \psi \right) \left( \frac{v}{s} \right)' \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 + \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] \\
\left( \frac{c}{x} \right)_i
\]

\[
\left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 + \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] \\
\left( \frac{c}{x} \right)_i
\]

\[
\left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 + \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] \\
\left( \frac{c}{x} \right)_i
\]

\[
\left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 + \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] \\
\left( \frac{c}{x} \right)_i
\]

\[
\left( \frac{c}{x} \right)_6 - \left( \frac{v}{s} \right)_v \left[ \frac{\mu v}{v} \left( \frac{c}{x} \right)_6 + \left( \frac{v}{s} \right)_v \left( \frac{c}{x} \right)_i \right] \\
\left( \frac{c}{x} \right)_i
\]
(\frac{w}{V} - \nu) \frac{w}{S} = \pm \frac{(\sqrt{3} - \nu)}{C_{1}/C_{L}} \theta' \left( \nu + \frac{w}{\lambda} \right) \left( \frac{w}{S} \right) - \\
\left( \nu + \frac{w}{\lambda} \right) \left( \frac{w}{S} \right)

This is Equation 24 of Reference 2. Reconsidering the fact \((C_{1}/C_{L})_{1} = (C_{1}/C_{L})_{S}\), the parameter \(\theta\) reduces to zero.

Thus,

\[(\frac{w}{V} - \nu) \frac{w}{S} = \pm \frac{(\sqrt{3} - \nu)}{C_{1}/C_{L}} \theta' \left( \nu \right) - \left( \nu \right) \frac{w}{\lambda} \left( \frac{w}{S} \right)\]

Where \((W/S)\)' involves all lift coefficients except that coefficient which is associated with the type of lift imposed on the foil in deriving the cavitation bucket. So,

\[(W/S)\)' = (C_{L})_{1} + (C_{L})_{S} + (C_{L}) + \frac{C_{1\text{eff}}}{C_{1}/C_{L}}\]

The term for residual lift, \(\frac{C_{1\text{eff}}}{C_{1}/C_{L}}\), is the lift associated with the sections in the normal plane.

This is now the general equation for the three dimensional foil cavitation bucket. It should be noted that buoyancy is not considered in the derivation. An illustration of this bucket is shown in Figure 3.

Basically the explanation is the same as that regarding Figure 1 but in this case cavitation will appear somewhere on the span as compared to somewhere on the section. In using this foil bucket to determine the non-
FIGURE 3  ILLUSTRATIVE EXAMPLE OF FOIL CAVITATION

BUCKET

15
cavitating regime of operation it must be noted that the bucket was derived using \( q' \) and not \( q \). So in order to determine the correct \( C_L \) and thus the correct operating speed, \( C_L \) must be multiplied by the factor \( q/q' \).

\[
C_L \text{ is not } \frac{W/S}{q} \quad \text{ but } \quad C_L = q/q' \frac{W/S}{q}
\]

So in summarizing Part I of this discussion it can be seen that the expression for the flap lift foil cavitation bucket is:

\[
\left( \frac{\Delta v \cdot \omega}{V} \right) \left( \frac{W}{S} \right) = \left( \sqrt{\frac{V}{S}} - \frac{\omega}{V} \right)^{\frac{1}{2}} \cdot \frac{1}{\left( \frac{\omega}{V} \right)} - \left( \frac{\omega}{V} \right) \cdot \left( \frac{\omega}{V} \right)
\]

which can be derived from the equation for section cavitation bucket,

\[
\sqrt{S} = \frac{V}{\omega} \left[ C_L \cdot (\omega - V) \right] + \left( \frac{\omega}{V} - \frac{\omega}{V} \right) \cdot \frac{V}{\omega}
\]

It is to the readers advantage to investigate the derivation of the Incidence Lift Cavitation Bucket discussed in Reference 2. The two equations, incidence lift and flap lift, are derived in quite similar manners. Being exactly the same except for the inclusion of the velocity distribution over the flap, which is in the \( \omega/V \) term and except for the definition of the \( (W/S)' \) term. The incidence lift starts with the basic definition of the cavitation number:
and through algebraic manipulation the equation for incidence lift, with zero pitch, reduces to

\[
\left( \frac{\alpha v}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]

(17)

It can be seen that the equations for incidence lift and flap lift reduce to the same quantities when the specific case of \( S = 0 \), and \( \alpha = 0 \) is used.

\[
\left( \frac{\alpha v}{V} - m \right) \left( \frac{\alpha w}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]

(Incidence Lift)

\[
\left( \frac{\alpha v}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]

\[
\left( \frac{\alpha v}{V} \right) \left( \frac{\alpha w}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]

(Flap Lift)

\[
\left( \frac{\alpha v}{V} \right) \left( \frac{\alpha w}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]

\[
\left( \frac{\alpha v}{V} \right) \left( \frac{\alpha w}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]

\[
\left( \frac{\alpha v}{V} \right) \left( \frac{\alpha w}{V} \right) = \frac{\sqrt{S - y}}{c_l/c_l}'
\]
At the point on the 1.25% chord station with $\delta = 0$ and $\alpha = 0$, the flapped foil becomes an incidence foil. This point can be mathematically computed to be:

\[
\frac{c_w}{v} \left( \frac{w}{c} \right)_n - \left( \frac{c_w}{v} \left( \frac{w}{c} \right) \right)_n = 0 = \frac{w}{c} \left( \frac{c_w}{v} \left( \frac{w}{c} \right) \right)_n + \left( \frac{c_w}{v} \left( \frac{w}{c} \right) \right)_n
\]

\[
0 = - \frac{w}{c} \left( \frac{c_w}{v} \right)_n + \left( \frac{c_w}{v} \left( \frac{w}{c} \right) \right)_n
\]

\[
\frac{w}{c} \left( \frac{c_w}{v} \right)_n = \left[ \frac{c_w}{v} \left( \frac{w}{c} \right) \right]_n
\]

\[
= \left[ \frac{0.869}{1.31} \right] \left[ \frac{2.990}{1.31} \right]
\]

\[
= 0.315 \frac{w}{c}
\]

Substituting and solving for $\sqrt{\delta}$ and then $V_k$:

\[
\frac{c_w}{v} \left( \frac{w}{c} \right)_n = \left( \frac{\sqrt{5} - 1}{\sqrt{5}} \right) \left( \frac{w}{c} \right)_n = 0
\]

\[
\left( \frac{\sqrt{5} - 1}{\sqrt{5}} \right) (0.315) = \frac{\sqrt{5} - 1}{1.31}
\]

\[
1.2256 = \sqrt{5}
\]

\[
= \sqrt{24.0112 + 1}
\]

\[
1.5021 = \frac{1392.995}{V_k^2}
\]

\[
V_k^2 = \frac{2774.339}{52.67 \cdot 0.315}
\]

\[
\frac{w}{c} = (52.67)^2 (1.876) (0.315)
\]

\[
= 1657.34 \text{ psi}
\]

\[
\frac{w}{c} = 1747.34 \text{ psi}
\]

18
FOIL CAVITATION BUCKET
AGEH FORWARD FOIL
DEPTH = 1 MAC
20% CHORD FLAP
INCIDENCE LIFT

FIGURE 4
The procedure for going from Equation 17 to Equation 18 is the basis for Reference 2 and is too lengthy to be covered in this note. Figure 4 shows the foil incidence bucket using the given section velocity distribution.

**PART II: REFERENCE LIFT COEFFICIENTS**

It should be noted that the foil lift coefficient, \( C_L \), does not appear anywhere in the foil cavitation bucket derivation for flap lift. But the foil lift coefficients are found in the terms \( W/S) \), and these foil loadings are related to those loadings of the section bucket through the terms \( (C_L)_{eff} \), \( \alpha \), and \( (C_L)_{eff} \). These six above mentioned terms shall be referred to as reference lift coefficients.

The incidence and pitch reference foil loadings are easily defined as:

\[
(W/S) = (C_L)_{eff} \alpha q \\
(W/S) = (C_L)_{inc} \alpha q
\]
Note that \( q' \) is employed and not \( q \). In order to determine a definition for \((W/S)_o\) its definition must be retraced. The \( W/S_o \) is loading due to the camber of the foil. \((W/S)_o\) is a component of \((W/S)\).

\[
(W/S)_o = \frac{(W/S)_0}{c_1/c_L} = \frac{c_{1\text{eff}}}{(c_1/^/c_L)} q'
\]

The reference section lift coefficients at the critical span stations are:

\[
(c_1)_1 = (c_L)_1 (c_1/c_L)_1 \\
\]

\[
c_{1\text{eff}} = \frac{c_{1\text{eff}}}{(c_1/c_L)_1} (c_1/c_L)_1
\]

In order to interpret the cavitation bucket as it is related to flap deflection another reference foil loading is required. This term shall be denoted \((W/S)_{ref}\), which has previously been present in the incidence lift case derivation, \((AG-18)\). The subscript "ref" is reserved for the product of a lift coefficient and the streamwise dynamic pressure.

\[
(W/S)_{ref} = \left[(c_L)_1 + c_{L_0} + (c_L)_1 q + (W/S)_B \right]
\]

This term is multipurpose in use in regards to the flap lift case. It takes into consideration all the factors of flap lift, namely the pitch, the Incidence, and the flap deflection and determines the operational zero flap deflection range in the bucket. This reference term can also be used to determine the validity of the \( c_L \) term used in the previous foil bucket equation derivation.
\[ c_L = c_{L\text{eff}} + c_{L\text{f}} + c_{L\text{s}} + c_{L\text{a}} \]

This ten provides a check on the entire theory of deriving the section and foil cavitation buckets. The lift coefficients due to pitch and incidence are obvious in their origin, whereas the residual lift coefficient, or that lift due to camber requires comment at this time.

If the effects of the pod are neglected the zero lift angles for incidence and section zero lift are equal.

\[ \alpha_0 = -\frac{c_{L\text{eff}}}{c_L} \]

Then \( c_{L\text{a}} \) must equal:

\[ c_{L\text{a}} = -c_{L\text{f}} \alpha_0 \]

(20)

\[ = c_{L\text{f}} \frac{c_{L\text{eff}}}{c_L \alpha} \]

For the best interpretation of the Lindsey, Stevenson, and Daley data available this relation results in

\[ c_{L\text{a}} = (2.49) (\frac{.330}{5.72}) \]

\[ = .144 \]

as compared to the value of .111 which was measured on the prototype. An explanation of the results can be seen in Appendix I. For a foil of fixed pitch and incidence the \( W/S \) \text{ref} term is a quadratic in \( V_k \) extending up to the cavitation bucket boundary.
In order to go directly from the given velocity distribution data to a foil cavitation bucket, equation 15 must be used. Recalling equation 15:

\[
\left( \frac{\Delta u}{V} - w \right) \frac{w}{S} = \pm \frac{\sqrt{S}}{\delta} \left( \frac{\Delta u}{V} \right) \left( \frac{w}{S} \right) - \frac{C_L}{C_L} \frac{w}{S} - w \left( \frac{w}{S} \right)'
\]

and noting that the U/S present in the equation does not include any buoyancy term,

\[
\frac{W/S}{S} = (W/S)_H + (W/S)_B
\]

\[
(W/S)_B = 0 \text{ in equation}
\]

\[
(W/S)_H = \text{U/S in equation for 3-D flap lift}
\]

Equation 15 can be further reduced by remembering the assumption of zero angle of attack. With \( \alpha = 0 \), \( (C_L)_\alpha = 0 \), and \( (C_L/C_L)_\alpha = 0 \) the parameter \( \int \) reduces to zero. Equation 15 can be simplified to equal:

\[
(22) \left( \frac{\Delta u}{V} - w \right) \left( \frac{w}{S} \right)_H = \pm \frac{\sqrt{S}}{\delta} \left( \frac{\Delta u}{V} \right) \left( \frac{w}{S} \right) - \frac{C_L}{C_L} \frac{w}{S} - w \left( \frac{w}{S} \right)'
\]

with

\[
\sqrt{S} = \frac{2044 + \gamma g h}{q}
\]

\[
\alpha = \frac{q}{q} \cos^2 \theta
\]

\[
(W/S)_H = (W/S)_1 + (W/S)_2
\]

\[
(W/S)_1 = (C_L)_{1q} \left( C_L + 1q \right)
\]

\[
(W/S)_2 = \left( \frac{C_L}{C_L} \right)_{1q} \left( C_L + 1q \right)
\]

\[
(W/S)_\alpha = (C_L)_{\alpha} \left( C_L + \alpha q \right)
\]
to the cavitation bucket boundary.

**PART III: CONSTRUCTION OF THE CAVITATION BUCKETS**

In order to graphically show the buckets and to in turn check the validity of the theory and graphs (See Part II) the flap lift case will be compared to the incidence lift case, one assumption will be made. The assumption being that the foil is operating at a zero degree angle of attack.

Equation 7 is the primary equation used in the construction of the section cavitation bucket. An expansion of the term $C_1'$ is necessary in order to see all working terms.

$$\sqrt{s} = \gamma = w[(C_1)_l + C_{1\text{eff}} + (C_1)_\infty]$$

It is appropriate at this point to introduce the depth effect factor. Because of free surface effects, which have a minor but not negligible effect, Panchenkov's depth effect factor from Figure 13 of Ref. 3 in a value of .923 to be used as an applied factor to all reference loadings. Using the velocity distribution data from Abbot and Von Doenhoff's *Theory of Wing Sections*, (condensed in Table 1) and Allen's velocity distributions over a flapped section, Figure 5, expressions in the linear form ($y=b+mx$) can be determined. These specific equations can be seen in Table 2 where $C_1$ has been equated to zero and unity. This then provides the data necessary to construct the section cavitation bucket, Fig. 6, for the AG(EH) forward foil. Access to this graph allows one to follow an alternate method of foil cavitation bucket construction. See Appendix II for this explanation.
Flap Basic Load Distribution, (AV/W)'

Chordwise Station, x/c

20% CHORD

LESS THAN 20°
FLAP BASIC LOAD DISTRIBUTION
SECTION CAVITATION BUCKET
AGEH FORWARD FOIL
20% CORD FLAP
FLAP LIFT

SECTION LIFT COEFFICIENT, C

FIGURE 6

26
where $C_{L_1}$ is derived from

$$ (23) \quad \frac{(W/S)_0 - (W/S)_{ref}}{q C_{\Delta \ell}} = 0 $$

at a design speed of fifty knots, and trimmed flaps

$$(W/S)_{ref} = (W/S)_D = 1435$$

This foil loading, being the design foil loading is then used in determining the value of $(C_L)_i$ for the flap lift case.

$$ \left[ (C_L)_i + C_{L_0} + (C_L)_{\alpha} \right] q_D + \left( \frac{N}{S} \right)_B = 1435 $$

$$ \left[ (C_L)_i + 0.923 (0.111) \right] (7100) = 1435 - 90 $$

$$(C_L)_i + 1.025 = 1347/7100 = 0.1894 $$

$$(C_L)_i = 0.0869 $$

The corresponding incidence angle is approximately:

$$ i = \frac{(C_L)_i}{C_L} = \frac{0.0869}{(0.923)(0.0438)} = 2.16^\circ $$

Since all variables and parameters are known for Equation 22 a foil cavitation bucket can be constructed. The equation was programmed and executed on a Hewlett-Packard 9810A calculator and the values are tabulated on Table III. A long hand check can be seen in Appendix III. Figure 7 shows the completed foil cavitation bucket. Also on the graph are the $(W/S)_{ref}$ and $(W/S)_D$ curves.
The cavitation bucket equation is a very powerful analytical tool. It allows the term \((C_L)_f\) to be used to compute a \((C_L)_S\) independent of the cavitation bucket equation. This then allows the schedule for flap deflection to be determined. The following equation can be employed when it is desired to compute a \((C_L)_S\).

\[
\left[ (C_L)_i + C_{L_0} + C_{L_\alpha} + C_{L_\delta} \right] q_D + (W/S)_B = 1435
\]
At this point in the discussion both the derivation and graphical representation of the foil cavitation bucket have been completed. How do these results compare to the existing incidence lift system? Figure 8 compares the limiting boundaries of the two different cavitation buckets. It can be observed that the flap lift case expands the already known incidence lift case boundaries. In order for the flaps to produce enough lift, \( g = 1 \), a flap deflection angle is required. In order to determine this angle Equation 24 is used.

\[
\delta_{lg} = \frac{W(S)_D - W(S)_{ref} + W(S)_B}{q C_{Lg}}
\]

This provides a concept that will be investigated in future notes; the idea of combining incidence and pitch angles along with flap deflection to provide the required lift with a minimum absolute value flap deflection. There also lies in the regime of the cavitation bucket a deflection angle of cavitation:

\[
\delta_{cav} = \frac{(W/S) - (W/S)^I}{q C_{Lg}}
\]

\[
= \frac{(W/S)_H + W(S)_B - W(S)_B - W(S)_B}{q C_{Lg}}
\]

Note: Positive deflection is a flap trailing edge downward.

From these last two relations plots can be made (See Figure 9, and Tables IV, V) and the two deflections can be compared. There are two points on the graph where \( \delta_{lg} = \delta_{cav} \). This could pose a grave problem since the deflection angles are within operational range. From Equations 24
RELATIONSHIP BETWEEN FLAP LIFT, INCIDENCE LIFT, AND \((W/S)'\) CURVES

FIGURE 8
and 2 \[ \frac{\delta t}{\text{ref}} \] can be seen that a positive deflection will move any point on the \( W/S \) curve toward the right, and any negative deflection will transfer the point to the left.

In summarizing what has been emphasized in this discussion, it can be seen that a set of very powerful equations have been derived. But the equations are not so complex that a person with minimal knowledge of cavitation cannot comprehend them.

**PART IV: OPTIMIZATION CONSIDERATIONS**

Configurations which will optimize the flap lift system will be covered in future notes when sufficient data on the system can be gathered.

Major discussions of importance are:

A. **Limited Flap Angles:**
   Determination and evaluation of the optimum flap angles when combined with angles of incidence and pitch, and which combination will result in least drag and incipient cavitation number.

B. **Smooth Water Cavitation Bucket:**
   Investigations in the distortions, expansions, reductions and extentions of the cavitation bucket which is produced in smooth water, with emphasis on various foil loadings, flap, incidence and pitch angles, and speeds.

C. **Rough Water Cavitation Bucket:**
   Investigation in the distortions, expansions, reductions, and extentions of the foil cavitation bucket which is produced in rough water with special emphasis on the effects caused by...
orbital velocity, and how to compensate for the varying wave heights.

D. Foil Drag
Resistance on the forward foil will be evaluated for the various combinations of flap, incidence, and pitch angles. These values, combined with the total drag on the pod, will give the total drag on the foil-pod configuration.

E. Hinge Moments
This will not be a consideration for flap lift.
If the derivation for $C_{L1}$ is incorporated into Equation 20
Recalling 20:

$$C_{L0} = C_{L1} \alpha_0 = C_{L1} \frac{C_{1\text{eff}}}{C_{1\alpha}}$$

all of the sources of error can be displayed.

$$C_{L0} = C_{1\text{eff}} \frac{C_{L1}}{C_{1\alpha} \text{ Proto}} \frac{C_{1\alpha} \text{ Proto}}{C_{1\alpha} \text{ Section}}$$

$C_{1\alpha} / C_{1\text{ Proto}}$ is the area under the incidence lift circulation distribution on the span. $C_{1\text{eff}}$ and $C_{1\alpha} \text{ Section}$ are distinct interpretations of the Lindsey, Stevenson, and Daley data. Thus there are three sources of error.

i) Neither the prototype nor the section experimental data is very reliable.

ii) There is an unestablished precision associated with the circulation distribution, particularly for partial span distributions and most particularly for sections not defined in the streamwise plane. Note that a section defined in a normal plane has its angle of attack reduced by the cosine of the sweep angle. If Equation 20 has this incorporated in it the value of $C_{L0}$ comes to within 5% of the measured prototype value.

iii) There is undoubtedly some pod influence, particularly at the foil root, which is not accounted for by treating incidence lift as a full chord, partial span flap case.
SOURCES OF ERROR IN $C_{L0}$

The most familiar manifestation of the prototype/theory $C_L$ discrepancy is the model/theory zero lift angle discrepancy which has been noted for years. Note that for the AGEH:

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_o = -\frac{C_{L0}}{C_{Li}}$</td>
<td>$\alpha_o = -\frac{C_{1_{\text{eff}}}}{C_{1\alpha}}$</td>
</tr>
<tr>
<td>$= -0.111/2.49$</td>
<td>$= -0.324/5.72$</td>
</tr>
<tr>
<td>$= 0.0496$</td>
<td>$= 0.0566$</td>
</tr>
<tr>
<td>$= 2.6 \pm 7% \text{ extreme}$</td>
<td>$= 3.24 \pm 13% \alpha$</td>
</tr>
<tr>
<td>$= 2.7 \text{ to } -2.4$</td>
<td>$= 3.66 \text{ to } -2.82$</td>
</tr>
</tbody>
</table>

It should be noted that $\alpha_o$ is invariant with depth, and $C_{1_{\text{eff}}}$ is subject to depth effect.

Applying the $\cos \Delta$ factor to the prototype, where $C_L$ is about that much higher than theory, would resolve this discrepancy. A similar discrepancy exists for DOLPHIN and FLAGSTAFF where the section is defined streamwise with a much smaller sweep.
A three dimensional foil cavitation bucket can be constructed by the to be mentioned procedure only after the construction of a precise section cavitation bucket has been completed.

The procedure is as follows,

Step : Choose 6 minimum of three valves of $\sqrt{S}$ and its corresponding $C_1$. Care must be taken when moving from upper to lower surface or vice versa since the ratio $C_1/C_L$ changes. Three valves are necessary since the foil bucket is not linear.

Step : Calculate corresponding valves of $V_K$ and $W/S$.

\[
S = 1 + \frac{P_0 - P_v}{q}
\]

\[
q = \frac{P_0 - P_v}{\sqrt{\frac{S}{1}}}
\]

\[
(0.9952)(1.6889)^2 V_K^2 = \frac{P_0 - P_v}{S - 1}
\]

\[
= \frac{P_at \times gh - P_v}{S - 1}
\]

\[
= 2116 + 64(9.33) = 72
\]

\[
V_K = \sqrt{\frac{2641.12}{2.8387}} \cdot \frac{1}{\sqrt{S - 1}}
\]

\[
= \frac{30.5}{\sqrt{S - 1}}
\]

and where $W/S$ is simply:

\[
W/S = C_1q' / C_1/C_L \delta
\]
Step 3: Now corresponding values of $V_k$ and $W/S$ can be determined. These can then be plotted with $W/S$ being the dependent variable and $V_k$ being the independent variable.
VERIFICATION OF FOIL CAVITATION

BUCKET PROGRAM

\[
\left( \frac{\Delta V}{V} - \frac{V}{V} \right)_{\text{M}} = \frac{\left( \sqrt{5} - 2 \right)}{5} \left( \frac{V}{V} + \frac{\Delta V}{V} \right) \left( \frac{\Delta V}{V} \right)
\]

\[
\left( \frac{\Delta V}{V} - \frac{V}{V} \right)_{\text{M}} = \left( \sqrt{5} - 2 \right) \times \left( \frac{V}{V} \right) \left( \frac{\Delta V}{V} \right)
\]

substituting values:

\[
(0.076 + 0.477) \left( \frac{\Delta V}{V} \right)_{\text{M}} = \left( \frac{1.2044 + (0.933) (0.04)}{1.869 (0.52)} \right) \times 1.31 (50^3)
\]

\[
(0.087 + 0.477) \left( \frac{\Delta V}{V} \right)_{\text{M}} = \left( \frac{1.2044 + (0.933) (0.04)}{1.869 (0.52)} \right) \times 1.31 (50^3)
\]

\[
-0.477 \left( \frac{\Delta V}{V} \right)_{\text{M}} = \left( \frac{1.2044 + (0.933) (0.04)}{1.869 (0.52)} \right) \times 1.31 (50^3)
\]

\[
\frac{\Delta V}{V} = \left( \sqrt{2} \times 1.15 \right) \left( \frac{0.477}{1.31} \right) 47.0 + 0.477(0.315)(47.0)
\]

\[
\frac{\Delta V}{V} = \left( \sqrt{2} \times 1.15 \right) 47.0 + 0.477(0.315)(47.0)
\]

\[
\frac{\Delta V}{V} = 451.34 + 712.62
\]

\[
\frac{\Delta V}{V} = 451.34 + 712.62
\]

\[
\frac{\Delta V}{V} = 2105.71 \text{ as compared to the HP value of 2105.82}
\]
DEFINITION OF $C_{i \text{eff}}$

There is a basic lift curve slope associated with the section camber for every section on the span given any angle of attack. At any angle of attack the camber is producing a lift, which shall be called from this point on in this series of notes, $C_{i \text{eff}}$, at every station on the span.

$$C_{i \text{eff}} = -C_1 \alpha_0$$

The coefficient of $C_1$ comes from Lindsey, Stevenson, and Daley, Reference 7. This value, multiplied by the section $C_1$, results in the value used for $C_{i \text{eff}}$ of .324.

In comparing the relationship between $C_{L0}$ and $C_{i \text{eff}}$, it shall be assumed that a zero lift angle is being used so there is no variation in lift and every station on the span will be acting along the zero lift angle.

$$-\alpha_0 = \frac{C_{i \text{eff}}}{C_{L0}} = \frac{C_{i \text{eff}}}{C_{L0}}$$

$$C_{L0} = \frac{dC_{i \text{eff}}}{d\alpha} (\alpha)$$

$$= \frac{dC_{i \text{eff}}}{d\alpha} \frac{C_0}{C_{L0}} \frac{C_1}{C_2} \frac{C_2}{C_3} \cos \Lambda$$

where $C_{L0} / C_{i \text{eff}}$ is the ratio of the areas under the lift curve slopes. These curves can be found in Reference 3.
The number resulting, .141, is larger than the prototype value of .111, which is in itself large since sweep was not taken into consideration, but by applying a factor equal to the cosine of the sweep angle (.817), the value of $C_{L0}$ can be reduced to near that of the prototype.

$$
C_{L0} = \frac{(0.81)(2.97)(0.324)}{(0.94)(2\pi)} \cos 35.2^\circ
$$

$$
= (0.141)(0.817)
$$

$$
= .115
$$
REFERENCES


SYMBOLS

a. All dimensions in ft./#/sec./rad. unless otherwise noted.
b. Parenthesis read "due to"; e.g. (CL), = CL due to flap deflection, $C_L \delta$.

c. Primes indicate normal to the quarter chord.

$C_L$ Foil Lift Coefficient, $L/qS$

$C_{L0}$ Residual Lift Coefficient

$C_{L1}$ Incidence Lift Curve Slope, $dC_L/d\theta$

$C_{L\alpha}$ Pitch Lift Curve Slope, $dC_L/d\alpha$

$C_{L\delta}$ Flap Lift Curve Slope, $dC_L/d\delta$

$C_1$ Section Lift Coefficient

$C_1$ $C_1$ at Zero Flap, $(C_1)_1 + C_{1eff}$

$C_{1f}$ $C_1$ for flap deflection, $C_{1\delta} d/\delta$

$C_1/C_L$ Measure of Spanwise Lift Distribution

$C_{11}$ Design Lift coefficient

$C_{1eff}$ See Appendix IV

h Depth

g Acceleration of Gravity

$\theta$ Incidence Angle

L Lift

$P_A$ Atmospheric Pressure (2116 psf)

$P_V$ Vapor Pressure (72 psf)

$q$ Dynamic Pressure, $\frac{1}{2} \rho V^2$

$\sqrt{s}$ $1 + \sigma$
\( V_k \) \hspace{1cm} \text{Speed in Knots}

\( \Delta v/V \) \hspace{1cm} \text{Local Velocity Distribution Due to Thickness}

\( \Delta v/V \) \hspace{1cm} \text{Local Velocity Ratio Increment Due to Camber}

\( \Delta v/V \) \hspace{1cm} \text{Local Velocity Ratio Increment Due to Flap}

\( \Delta v/V \) \hspace{1cm} \text{Basic Load}

\( \Delta v/a/V \) \hspace{1cm} \text{Local Velocity Ratio Increment Due to additional Load, Angle of Attack and/or Flap Deflection}

\( W/S \) \hspace{1cm} \text{Hydrodynamic Foil Loading}

\( w/s \) \hspace{1cm} \text{Section Foil Loading}

\( W/S \) \hspace{1cm} \text{Buoyant Foil Loading, B/S}

\( W/S \) \hspace{1cm} \text{Design Foil Loading}

\( W/S \) \hspace{1cm} \text{Pitch Foil Loading, } C_L = \theta q'

\( W/S \) \hspace{1cm} \text{Incidence Foil Loading, } C_{Li} q'

\( W/S \) \hspace{1cm} \text{Flap Foil Loading, } C_{Ls} q'

\( W/S \) \hspace{1cm} \text{Reference Foil Loading}

\( \alpha \) \hspace{1cm} \text{Angle of Attack}

\( S \) \hspace{1cm} \text{Flap Deflection, Positive Nose Up}

\( f_1 \) \hspace{1cm} \text{Spanwise Load Distribution Parameter, } \frac{C_1/C_L}{C_1/C_L - 1}

\( f_\alpha \) \hspace{1cm} \text{Spanwise Load Distribution Parameter, } \frac{C_1/C_L}{C_1/C_L - 1}

\( \rho \) \hspace{1cm} \text{Density, } 1.9905 \text{ lbf sec}^2/\text{ft}^4

\( \sigma \) \hspace{1cm} \text{Cavitation Parameter, } (P_A - P_V + \rho gh)/g

\( \psi \) \hspace{1cm} \text{Spanwise Load Distribution Parameter, } \int \left[ \Delta v/a/V = \Delta v/V \right]

\( \Delta \) \hspace{1cm} \text{Sweep Angle}

\( \psi \) \hspace{1cm} \text{Chordwise Velocity Distribution}

\( V/V \) \hspace{1cm} \text{Chordwise Lift Distribution Parameter}

\( \Delta \) \hspace{1cm} \text{A Flap Chordwise Lift Distribution Parameter}

48
### TABLE I

**CAVITATION PARAMETERS**

<table>
<thead>
<tr>
<th>Station % chord</th>
<th>1.25</th>
<th>2.5</th>
<th>5.0</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.380</td>
<td>1.284</td>
<td>1.201</td>
<td>1.098</td>
<td>1.117</td>
<td>1.131</td>
<td>1.136</td>
<td>1.123</td>
</tr>
<tr>
<td>$\Delta \alpha / \alpha$</td>
<td>1.346</td>
<td>0.970</td>
<td>0.686</td>
<td>0.196</td>
<td>0.160</td>
<td>0.131</td>
<td>0.103</td>
<td>0.076</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.620</td>
<td>0.399</td>
<td>0.301</td>
<td>0.10</td>
<td>-0.027</td>
<td>-0.71</td>
<td>-1.38</td>
<td>-0.477</td>
</tr>
</tbody>
</table>

| Note: Upper numbers and signs denote upper surface, lowers denote lower surface |

16-(.390)08 Section  
1 MAC (9.33) Depth  
20% Chord Flap  
$q = 2.8387 \frac{V^2}{k}$  
$q' = 0.668 \frac{V^2}{k}$  

$$ \gamma = \frac{v}{V} \pm \frac{\Delta \alpha / \alpha}{1 - \frac{C_{l,eff}}{C_{l} / C_{L}} \Delta \alpha / \alpha} $$

$$ \omega = \int \left[ \frac{\Delta \alpha / \alpha - (\Delta \alpha / \alpha)_f}{1 - \frac{C_{l,eff}}{C_{l} / C_{L}} \Delta \alpha / \alpha} \right] $$

$$(C_{l} / C_{L})_f = (C_{l} / C_{L})_1 = 1.31$$

$$(C_{l} / C_{L})_{ieff} = 0.71$$

$$ s_a = -0.109 $$

$$ s_a = 0.240 $$

$$ C_{l,eff} = 0.324 $$
CALCULATION TABLE I

\[ \gamma^2 = \varphi \left[ \delta \right] \pm \left( \frac{1}{a} \varphi \right) \Delta \gamma \]

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\gamma^2$</th>
<th>$\Delta \gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>$0.670 + (0.620)(0.4129) + (1.346-0.620)\Delta \gamma = 0.925 + 0.726\Delta \gamma$</td>
<td>$0.925 + 0.726\Delta \gamma$</td>
</tr>
<tr>
<td>2.5</td>
<td>$0.818 + (0.439)(0.4129) + (0.970 - 0.439)\Delta \gamma = 0.999 + 0.531\Delta \gamma$</td>
<td>$0.999 + 0.531\Delta \gamma$</td>
</tr>
<tr>
<td>5.0</td>
<td>$0.919 + (0.301)(0.4129) + (0.686 - 0.301)\Delta \gamma = 1.04 + 0.385\Delta \gamma$</td>
<td>$1.04 + 0.385\Delta \gamma$</td>
</tr>
<tr>
<td>40</td>
<td>$1.098 + (0.010)(0.4129) + (0.196 + 0.010)\Delta \gamma = 1.102 + 0.186\Delta \gamma$</td>
<td>$1.102 + 0.186\Delta \gamma$</td>
</tr>
<tr>
<td>50</td>
<td>$1.117 + (-0.027)(0.4129) + (0.160 + 0.027)\Delta \gamma = 1.105 + 0.187\Delta \gamma$</td>
<td>$1.105 + 0.187\Delta \gamma$</td>
</tr>
<tr>
<td>60</td>
<td>$1.131 + (-0.071)(0.4129) + (0.131 + 0.071)\Delta \gamma = 1.101 + 0.202\Delta \gamma$</td>
<td>$1.101 + 0.202\Delta \gamma$</td>
</tr>
<tr>
<td>70</td>
<td>$1.136 + (-0.138)(0.4129) + (0.103 + 0.138)\Delta \gamma = 1.079 + 0.241\Delta \gamma$</td>
<td>$1.079 + 0.241\Delta \gamma$</td>
</tr>
<tr>
<td>80</td>
<td>$1.123 + (-0.477)(0.4129) + (0.076 + 0.477)\Delta \gamma = 0.926 + 0.553\Delta \gamma$</td>
<td>$0.926 + 0.553\Delta \gamma$</td>
</tr>
<tr>
<td>11.25</td>
<td>$1.380 - (0.620)(0.4129) + (-1.346 + 0.620)\Delta \gamma = 1.156 - 0.726\Delta \gamma$</td>
<td>$1.156 - 0.726\Delta \gamma$</td>
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<tr>
<td>12.5</td>
<td>$1.284 - (0.439)(0.4129) + (-0.970 + 0.439)\Delta \gamma = 1.125 - 0.531\Delta \gamma$</td>
<td>$1.125 - 0.531\Delta \gamma$</td>
</tr>
<tr>
<td>15.0</td>
<td>$1.201 - (0.301)(0.4129) + (-0.686 + 0.301)\Delta \gamma = 1.092 - 0.385\Delta \gamma$</td>
<td>$1.092 - 0.385\Delta \gamma$</td>
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### TABLE III
RESULTS OF FOIL CAVITATION
BUCKET EQUATION

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<th>L2.5</th>
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Add 90 psf to all values
CALCULATION TABLE IV

\[ \delta^o_{1g} = \frac{\text{W/S})_D - \text{W/S})_{\text{ref}}}{qC} \]

<table>
<thead>
<tr>
<th>( V_k )</th>
<th>( \delta^o_{1g} )</th>
<th>( V_k )</th>
<th>( \delta^o_{1g} )</th>
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**CALCULATION TABLE V**

\[ \delta_{cav}^0 = \frac{W/S)_H - W/S)'}{q'c_LS} \]

<table>
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<th>( v_k )</th>
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<th>( v_k )</th>
<th>( \delta_{cav}^0 )</th>
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</table>
CALCULATION TABLE VI

\[ W/S)_{ref} = \left( C_L \right)_{i} + C_{L0} + C_{LS} + C_L \]

\[ W/S)' = \left( C_L \right)_{i} + C_{LC} + C_{leff}/(C_L/C_L) \]

<table>
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<th>( W/S)' )</th>
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