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DESIGN CRITERIA FOR DETAIL FOIL DESIGN

(FULLY SUBMERGED, SUBCAVITATING FOILS)

HYDROFOIL ADVANCED DEVELOPMENT PROG. NSRDC

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FOREWORD

The following material was prepared as one section of a comprehensive study of the criteria applicable to the design of hydrofoil ships. It is published separately, and in advance of other parts of the study, for convenience in use and to facilitate an early review.

The arrangement reflects the focus of the discussion on determination of those geometrical characteristics which appear to the observer and which are delineated on the plans, thus constituting the design. The criteria which form the bases for design decisions are the relations between design characteristics and foil performance. Inevitability, conflicting influences are at work and compromises are required as is frequently noted. Ultimately recourse must be had to the statement of requirements, derived from the intended mission, in order to establish the relative importance of various aspects of performance and thus permit a judgement based on determinable tradeoffs.

A certain amount of design data has been presented, primarily by way of example. No attempt at completeness has been made; it was not the intention to prepare a design data handbook. Rather it is hoped that a framework is provided to which the growing design data base can be related.

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III.2.1.1 INTRODUCTION

When the general arrangement of the strut/foil system has been selected, and the load to be carried on each foil has been established, the detailed design of the individual foils can be undertaken. This involves the determination of the area of the foil, of its proportions or aspect ratio, and other characteristics of the planform such as sweepback and taper and tip shape, as well as the section profile and, perhaps, twist.

Selection of these characteristics will be influenced by prior decisions concerning the foil arrangement which, even at an early date, must be made with due regard for their effects on these later design details. In addition interactions occur which make many of the features interdependent. Consequently no one characteristic can be established firmly until the effect on others has been evaluated. This makes it difficult to establish a design procedure of general applicability. The following suggestions are offered, for what help they may be. The one certainty is that each decision will have to be reviewed to resolve conflicts.

Because the foil area so profoundly effects several aspects of performance, and because any increase leads to additional difficulty in retraction and docking, its determination must be an early order of business. The effects of taper and sweep are so interdependent they must be considered together. A first cut at the maximum section thickness ratio is probably next in order, followed by a determination of the aspect ratio. The section design can then be refined, along with a determination regarding the necessity for twist.

In the following subsections each of these characteristics is discussed and the applicable criteria are developed in considerable detail.

A summary of the important features is embodied in the table on the following page.

CHARACTERISTIC:	FOIL AREA (LOADING, $\frac{L}{S}$)	ASPECT RATIO	SWEEPBACK
CRITERIA:	1. DRAG a. Achieve Minimum Drag at "Cruise" Speed b. Limit Drag at Takeoff c. Limit Drag at Top Speed 2. CAVITATION a. Limit Loading at Top Speed b. Provide Acquate Loading Range at Top Speed c. Avoid Leading Edge Cavitation at Minimum Foilborne Speed d. Manufacturing Tolerances 3. TAKEOFF a. Provide Adequate Lift for Takeoff at Desired Speed 4. SIZE a. Ease of Retraction b. Obstruction to Maneuvering and Docking	1. DRAG a. Increase Aspect Ratio to Reduce Foil Induced Drag b. Penalty if Additional Strut/Pod is Required 2. STRENGTH a. Increase of Bending Moment With Increased Aspect Ratio b. Decrease of Bending Strength With Increased Aspect Ratio c. Reduced Torsional Stiffness With Increased Aspect Ratio -- Possible Hydroclastic Effects 3. SIZE a. Ease of Retraction b. Obstruction to Maneuvering and Docking	1. CAVITATION a. Increased Allowable Thickne Increase of Sweepback b. Affects Spanwise Load Distri 2. DRAG a. Affects Spanwise Load Distri and Induced Drag 3. SHED FLOTSOM
RELEVANT REQUIREMENTS:	Maximum Speed Speed for Maximum Range Speed for Maximum Endurance Minimum Takeoff Speed Sea State & Riding Qualities	Takeoff Speed Cruising Speed Minimize Structural Weight	Minimize Power Minimize Structural Weight
RELATED CHARACTERISTICS:	Overall Foil Configuration Planform: a. Aspect Ratio b. Taper c. Sweepback Lift Control Means Foil Section	Taper Sweepback Foil Structure Foil Material	Aspect Ratio Taper

SUMMARY

	TAPER	FOIL SECTION (THICKNESS RATIO, PROFILE, MEAN LINE)	TWIST
ss with bution bution	USE TAPER TO: 1. Improve Strength a. Increased taper reduces foil bending moment over whole of semi-span. (Affected by twist and camber variation.) b. Increased taper increases foil root bending strength. c. Increased taper reduces foil bending strength for fixed root strength. Taper limited to 0.2 with geometrically similar structural sections. d. Increased taper reduces twisting moments. Probably increases critical torsion-bending flutter speed. 2. Improve Spanwise Loading, Reduce Induced Drag a. For unswept foil, taper ratio of 0.45 is optimum. b. With increased sweepback, more taper is required. c. Effect of taper (and sweep) can be modified by twist and camber variations. 3. Reduce Weight of Structure a. Use of taper will permit easier matching of strength to loads, hence eliminate excessive material. TAPER IS LIMITED BY: 1. Cavitation a. Increased taper increases section lift coefficient near the tip. (Effect is modified by twist and camber variation.) OTHER CONSIDERATIONS: 1. Control a. In conjunction with sweepback, taper affects pitching moment with foil lift coefficient.	1. CAVITATION a. Maximum Loading at Top Speed b. Minimum Foilborne Speed 2. MAXIMUM C_L a. Takeoff Speed b. Drag at Takeoff 3. STRENGTH a. Section Modulus	USE TWIST TO MODIFY SPANWISE LOADING TO COUNTERACT UNWANTED EFFECTS OF: a. Sweepback and Taper b. Proximity to the Free Surface c. Downwash From Other Foils d. Spanwise Variations of Camber e. Elastic Deformation
	Minimize Structural Weight Minimize Power Control Cavitation Damage	Design Speed Minimum Cruise Speed Minimum Takeoff Speed Sea State	Control Cavitation Damage & Noise
	Sweepback Aspect Ratio Twist Spanwise Section Variation	Foil Area Planform Strut Arrangement Arrangement of Pods & Nacelles Control Surface Design	Sweepback Taper Aspect Ratio Section Geometry

III.2.1.2 FOIL AREA

The area, S, of a given foil may be expressed as the product of the span, b, and the mean chord, c_m. Alternatively the expression $S = \frac{b^2}{AR}$ is used. The span and aspect ratio, $AR = \frac{b}{c_m}$, each has a significant influence on performance, as will be described later. But there are vital aspects of performance which depend primarily on the area, and the resulting lift coefficient. It appears productive, therefore, to discuss this characteristic by itself.

Given the lift to be developed on one foil, the area of the foil must be such as to satisfy several requirements. First of all, takeoff must be possible at the specified speed. Secondly, subcavitating foils must be free of cavitation under almost all conditions of operation. Thirdly, the drag must be a minimum consistent with the above criteria. In addition, structural requirements and propulsion system characteristics may influence the choice of foil area. Finally, practical considerations of docking and retraction put a limit on the tolerable size of foils.

Which criterion will govern the foil size depends, as will be seen, on the speed requirements and their relative importance. Thus, if minimization of power at maximum speed is critical, a lift coefficient of about 0.4 at that speed will be sought. Very probably a lower lift coefficient would be required at maximum speed to avoid cavitation. For speeds approaching 50 knots lift coefficient values of the order of 0.2 have had to be used.

With a foil designed for maximum speed, the takeoff speed and the minimum flying speed may be unacceptably high. The hull drag at the hump before takeoff may be so great that the power required for takeoff is considerable above that required at top speed. Such a foil, while free of cavitation at the design speed, may suffer from leading edge cavitation at lower speeds.

Consequently if lower takeoff or cruise speed is required, and especially if minimum drag and power are required at a low cruise speed, a further increase of area will be necessary. The resulting drag penalty at maximum speed will be somewhat compensated by a larger allowable thickness and consequent allowable aspect ratio.

A more detailed discussion of these considerations follows.

III.2.1.2.a Maximum Lift Coefficient

The minimum takeoff speed is absolutely limited by the maximum lift coefficient which can be achieved and by the foil area. Thus the foil area must be at least

$$S = \frac{L}{\frac{1}{2}\rho \; V^2_{to} \; C_L}$$

For a given lift, L, and takeoff speed, V_{to}, the required area is thus determined by the maximum attainable lift coefficient.

The overall foil lift coeficient will be less than the peak section lift coefficient, depending primarily on the planform, as discussed in Section III.2.1.3. The maximum attainable sectin lift coefficient will, in turn, depend on section thickness and camber; see Section III.2.1.5. In addition, the usable lift coefficient at takeoff may be less than the maximum of which the isolated foil is capable, for several reasons. For example, if flaps are used for lift control on a fixed foil so that the whole craft must be pitched up to increase the foil angle of attack, it may not be possible to utilize anywhere near the maximum foil capability. Mechanical restrictions might also prevent the achievement of the full potentiality of an all-moveable foil. Furthermore, some lift margin must be available for control at takeoff.

The lift/drag ratio of the foils deteriorates very rapidly as the lift coefficient approaches its maximum value. As a result the foil drag at takeoff would be substantially larger than at higher foilborne speeds, if maximum C_L were required for takeoff. The capability of the propulsion system to provide the necessary thrust would, therefore, be a critical factor. An increase in foil area, beyond the minimum necessary to lift the ship, may be required on this account.

III.2.1.2.b Cavitation

The erosive effects of prolonged cavitation render the avoidance of cavitation a matter of highest priority. The alternative is a continuing struggle to repair damage to the foil surfaces and coatings.

Cavitation is a source of noise and thus a prime factor in sonar detectability. It need not be extensive or damaging to be audible. On the other hand, the noise produced may well be obscured by that due to propeller cavitation. Hence the influence of acoustic requirements on foil design will depend on the possibility of eliminating other sources, as well as on the mission requirements of the craft.

Cavitation over a limited extent of the foil does not usually have a noticeable effect on the performance of the foil. In some cases cavitation near the leading edge may increase the lift coefficient but the effect on drag is always detremental. More extensive cavitation leads to a substantial loss of lift and increase of drag. This may be important for lower speed operation, or at takeoff, with a foil designed primarily for high speed flight.

The analysis of foil design to achieve freedom from cavitation has been extensively treated. The basic consideration is, of course, that cavitation occurs when the pressure in the flow over the foils is reduced to the vapor pressure, approximately. At a given depth of submergence, this occurs when the velocity of the flow -- relative to the foil -- is sufficiently accelerated with respect to the craft velocity. Several influences contribute to the acceleration of the flow and to the concomitant pressure reduction.

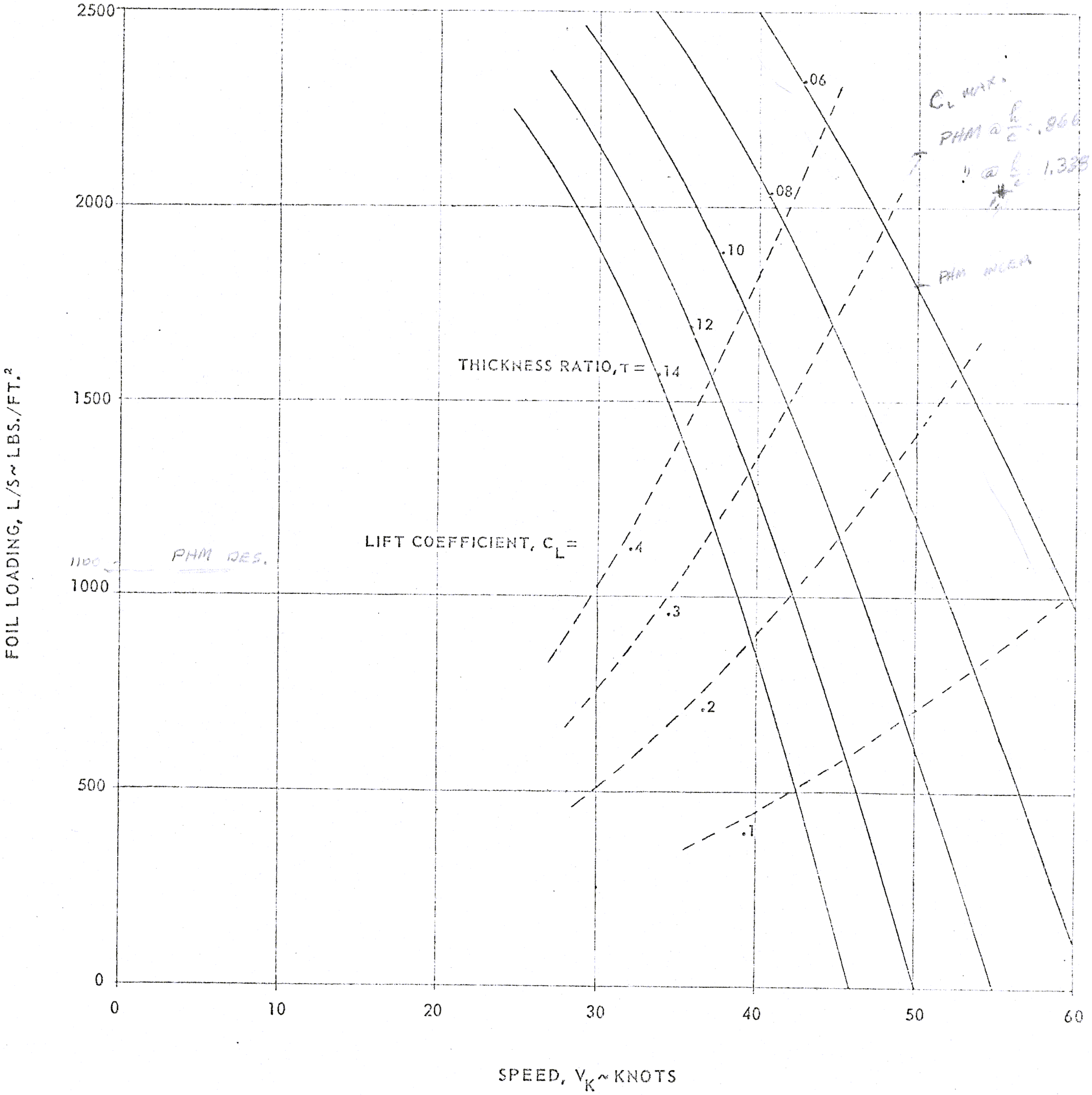
The final effect of foil design to avoid cavitation is frequently expressed in terms of the ratio of foil lift to foil area. A thin foil can be cambered to produce a nearly uniform pressure difference and thus provide the needed lift. Since half the pressure difference appears as an increase on the lower surface and half as a decrease on the upper surface, an absolute limit of lift is imposed amounting to something over 4000 pounds per square foot for a foil near the water surface. For reasons which are set forth below only a fraction of this can be achieved practically.

1. EFFECT OF THICKNESS

A substantial foil thickness is required in order to accommodate the structure necessary to support the lift and to transmit this force into the struts. As a consequence the flow is further accelerated and the pressure reduced on both the top and the bottom of the foil. As a result the pressure difference and the loading which can be tolerated without cavitation are reduced. This is clearly shown in Figure III.2-1 which indicates the maximum loading which can be carried on a foil of given thickness at a given speed. The effect of thickness varies as the square of the speed and is the principal reason for the increasing difficulty in avoiding cavitation at higher design speed. An increase of area will permit some reduction in thickness, for equal strength, thus contributing to a lower thickness/chord ratio and helping to alleviate the associated pressure reduction.

2. EFFECTS OF SPEED AND LOAD VARIATIONS

By careful design to achieve a nearly uniform pressure reduction on the upper surface, an approach can be made to an optimum foil for a given speed and loading. At any other speed the angle of attack -- or the flap deflection angle -- must be changed to maintain the required lift. As a result the pressure distribution is altered and the minimum pressure is reduced. Even at the design speed, when operating in waves, it will not be possible to maintain the design angle of attack and flap deflection. It is necessary, then, to provide a cavitation margin under steady state design conditions to avoid excessive intermittent cavitation in waves and to assure cavitation-free operation at off-design speeds. This problem is discussed in greater detail in Section III.2.1.5 where the design of the foil section is treated.



MAXIMUM CAVITATION-FREE LOADING FOR FOILS OF VARIOUS THICKNESS RATIOS. NACA 66 SECTION (DTMB MODEL) WITH a=0.8 MEANLINES AT MID-ENVELOPE OPERATION AT 7½ FEET SUBMERGENCE. (DERIVED FROM FIGURE III.2-18)

FIGURE III.2-1

3. PLANFORM EFFECTS

It is well known that an elliptical foil outline, or planform, providws an optimum spanwise load distribution. When combined with a suitable section design, such a foil can assure a favorable pressure distribution over the foil surface. Because of the high cost of constructing a foil with elliptical outline, however, it is much more common to use trapezoidal semi-span planforms (often referred to as straight-tapered), perhaps incorporating sweepback, in an attempt to approximate the performance of the elliptical foil. On such a foil the pressure distribution is distorted in a manner depending on the aspect ratio, the taper and the sweepback.* As a result, at a given loading and speed, the minimum pressure will be lower and cavitation will occur earlier than on an elliptical foil. Additional foil area will be required on this account. Further discussion of this point will be found in Section III.2.1.3.

*See Thwaites, Reference 1, Chapter VIII.

4. FREE SURFACE EFFECTS

The pressure distribution on the foil is modified by the proximity to the water surface. Since the submergence of the foil will vary, some allowance must be made to accommodate the resulting variation of the minimum pressure.

Very severe changes of pressure distribution occur when the foil approaches so close to the surface that air penetrates into low pressure regions. Such ventilation always occurs after the foil broaches the surface and reenters. The resulting effects on foil lift appear to be unavoidable if broaching is to be accepted. On the other hand, any attempt to prevent all broaching would produce unacceptable craft motion, hence broaching is not considered in the determination of foil area.

5. INTERFERENCE EFFECTS

In the wake of a forward foil, there will be a downwash between the trailing tip vortices and an upwash outboard thereof. The angle of attack on an after foil is reduced in the downwash region and increased in the upwash. The result is a distortion of the spanwise load distribution and of the pressure distribution on the foil. By incorporation of suitable twist these effects can be compensated at the design speed, but there would remain a distortion at other speeds or while turning. Such interference effects must be considered when selecting the area(s) of the after foil(s).

6. EFFECT OF MANUFACTURING ERRORS

The achievement of a desirable pressure distribution is dependent on obtaining a very fair, smooth and accurate surface on the foils. Unfairness, of the order of magnitude common in shipyard practice, will cause cavitation to occur at a lower speed than that predicted on the basis of theory or of wind or water tunnel tests on very carefully fabricated models. Only limited progress has been made in the correlation of possible types and degrees of roughness and unfairness with the extent of performance deterioration. Consequently the allowance to be made or, conversely, the accuracy of manufacture to be specified is very difficult to establish.

III.2.1.2.c Drag

Within the normal range of the lift coefficient and for plain foils, the drag of the foil includes two principal components, the induced drag and the parasitic profile drag. Thus

$$D = D_i + D_p$$

where D is the total foil drag

D_i is the induced drag

D_p is the profile drag

It is more usual to express the equation in terms of nondimensional coefficients:

$$C_D = \frac{D}{qS} = \frac{D_i}{qS} + \frac{D_p}{qS}$$

$$= C_{Di} + C_{Dp}$$

where $q = \frac{1}{2}\rho V^2$ is the stagnation pressure of the flow

S is the area of the foil.

It is shown* that the induced drag is proportional to the square of the lift so that

$$C_{Di} = \frac{C_L^2}{\pi e AR}$$

where $C_L = \frac{L}{qS}$ is the lift coefficient and e is an efficiency factor which has the value 1.0 when the spanwise load distribution is elliptical and will be slightly less than 1.0 for tapered foils. The profile drag coefficient is not exactly a constant, being generally a minimum at the design lift coefficient for the foil and including an increment which is also proportional to the square of the lift.** If we assume that this increment is absorbed into the factor e, we can derive a useful expression for the lift/drag ratio in the following form.

*Reference 9, pp. 202 and 206

**Thwaites, p. 329

$$\frac{D}{L} = \frac{C_D}{C_L} = \frac{C_L}{\pi e AR} + \frac{C_{D_o}}{C_L}$$

where C_{D_o} is the minimum profile drag coefficient.

By expressing the aspect ratio as

$$AR = \frac{b^2}{S}$$

and

$$C_L = \frac{L}{qS}$$

the equation above can be written in the alternative form

$$\frac{D}{L} = \frac{L}{\pi e} \cdot \frac{1}{qb^2} + \frac{C_{D_o}}{L} \cdot qS$$

which shows that, for a given lift and speed, the induced drag depends only on the span and varies inversely as the square of the span. The profile drag varies directly as the area, since the minimum profile drag coefficient is nearly constant. The problem in design is to achieve a large span with a small area, since both these trends are detrimental to the strength of the foil. There is also, as has been noted, a minimum area required to avoid stall.

These relations are plotted in Figure III.2-2 for what are considered to be conservative values of e and C_{D_o}. The figure shows the lift/drag ratio as the ordinate, against

$1/\sqrt{C_L} = V\sqrt{\frac{\rho S}{2L}}$ as the abscissa, for constant values of the aspect ratio. A scale of C_L is shown at the top of the figure. Dotted contours have been drawn for constant values of

$bV\sqrt{\frac{\rho}{2L}} = \sqrt{\frac{AR}{C_L}}$, hence constant induced drag for a given lift.

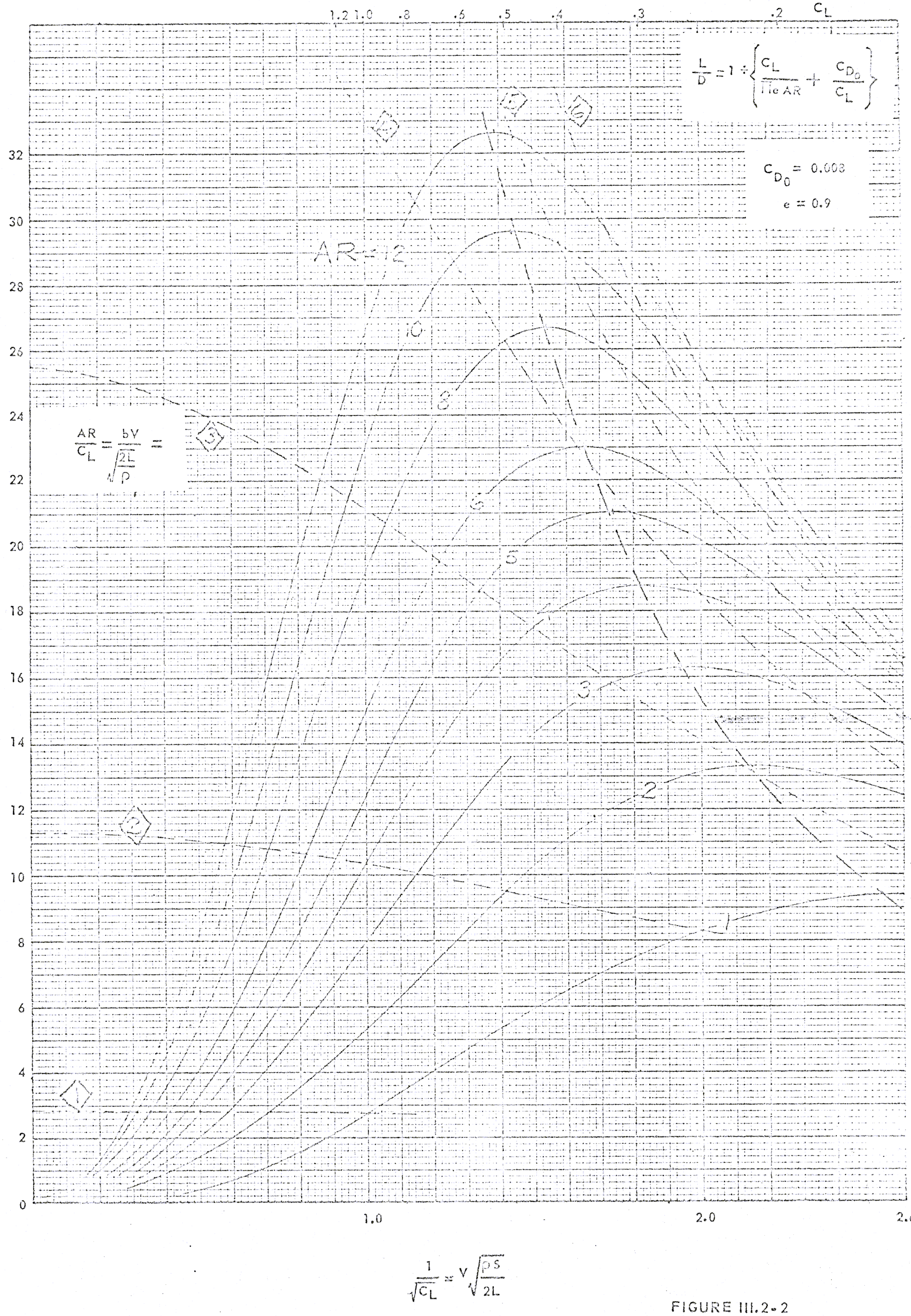


FIGURE III.2-2

For a given ship, since the aspect ratio and the ratio $\frac{\rho S}{2L}$ remain constant, the solid contours show the variation of L/D against a linear scale of speed. At low speed the lift coefficient is high, the induced drag is dominant and the lift/drag ratio deteriorates. At high speed the lift coefficient is low and the profile drag predominates. At an intermediate speed* the drag is a minimum and the lift/drag ratio an optimum.

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*At the speed where $D_i = D_\phi = \frac{D}{2}$

The minimum drag is evidently strongly dependent on the aspect ratio, which should be as high as possible. The speed at which the drag is a minimum is primarily a function of the loading, $\frac{L}{S}$, - thus of the foil area, S - and to a lesser extent of the aspect ratio. In fact the optimum lift coefficient is

$$\left.C_L\right)_{OPT} = \sqrt{\pi \; e \; C_{D_o} \cdot AR}$$

so that

$$\left.V^2\right)_{OPT} = \frac{\frac{2}{\rho}\frac{L}{S}}{\sqrt{\pi \; e \; C_{D_o} \cdot AR}}$$

An equivalent expression is

$$\left.V^2\right)_{OPT} = \frac{2}{\rho}\sqrt{\frac{L/S \cdot L/b^2}{\pi \; e \; C_{D_o}}}$$

The locus of optimum lift coefficients is shown on Figure III.2-2.

Since the maximum attainable lift coefficient is substantially independent of the aspect ratio, the ratio of the speed for minimum drag to the takeoff speed will vary with the aspect ratio. For practical values of the aspect ratio, and assuming a maximum lift coefficient of 1.0, this speed ratio will be in the range from 1.4 to 1.7. Thus an attempt to minimize the drag at maximum speed will result in a takeoff speed of 60% to 70% of the maximum speed; that is 30 to 35 knots for a 50 knot top speed. Furthermore the power required for takeoff would almost certainly exceed that for the desired maximum speed. Depending on the hull design the actual peak power may well be required below takeoff speed. In effect, if sufficient power is provided for takeoff without a special boost device, the maximum speed attainable will be above the speed for minimum drag.

It has been the practice to increase the foil area and reduce the takeoff speed, hence also the power at takeoff, until a balance is achieved between the power required for takeoff in rough water and that at the desired maximum speed. Due account must be taken of the variation of propulsive coefficient and engine fuel rate with speed. This represents a minimum power level for a given weight and maximum speed. It must be verified that the resulting foil area is sufficient from the cavitation point of view.

Design to this criterion establishes a minimum flying speed, and a minimum takeoff speed. If either of these must be further reduced, then a further increase of foil area may be necessary. An alternative is the use of flaps to increase the maximum lift coefficient. This will reduce the foil drag at high lift coefficients and, by permitting takeoff at a lower speed, will also reduce the hull drag and the power required for takeoff.

III.2.1.3 FOIL PLANFORM

We consider here the proportions and form of the individual foils. The individual foil planform characteristics must of course be consistent with the features of the overall configuration and certain compromises are to be anticipated.

The predominant features of the planform are the aspect ratio, sweepback and, taper ratio.

An unswept, elliptical planform has been shown[1] to give a minimum of the induced drag, for given span, under free stream conditions. For such a foil, the spanwise load distribution is elliptical and the section lift coefficient and the chordwise distribution are spanwise uniform. For these reasons the elliptical foil is used as a standard for comparison of performance with other foils. Some airplane wings have been built with elliptical planforms, but the lower cost of construction has led to the widespread use of polygonal planforms. The following discussion will refer mostly to planforms with trapezoidal semi-span panels. The effects of limited rounding of the tips will also be considered, but no attempt will be made to treat the limitless variety of forms which might be used.

III.2.1.3.a Aspect Ratio

The aspect ratio is defined as the ratio of the span to the mean geometric chord of the foil. This is equivalent to

$$AR = \frac{b^2}{S}$$

where

b is the span

S is the area of the foil.

The aspect ratio, in conjunction with the area which determines the lift coefficient, has a controlling influence on the drag characteristics of the foil. At the same time the aspect ratio, along with the area, the taper ratio and the thickness, significantly affects the strength of the foil. For minimum drag the aspect ratio should be as high as possible; for maximum strength, as low as possible. Thus a compromise is necessary. A more detailed discussion follows.

- DRAG

It was shown in Section III.2.1.2.c that the area of the foil governs the profile drag while the span controls the induced drag. Variation of either in the sense required to reduce the drag increases the aspect ratio. Thus a higher ratio is conducive to an improved lift/drag ratio, as is evident in Figure III.2-2.

One way to achieve a higher aspect ratio is to increase the number of supporting struts, thus reducing the maximum bending moment. The reduction in foil drag which results must be balanced against the added strut and junction drag. Thus a substantial increase of aspect ratio must be achieved before a net gain is realized.

It may also be remarked that, if a single strut supporting a foil is made wholly steerable, then the substitution of two struts introduces considerable mechanical complication.

When two struts are to be fitted for the main foil, or foils, it is evident that the use of a single foil will permit a larger span, a higher aspect ratio and a better lift/drag ratio than may be obtained with split foils. The only real tradeoff is the increased difficulty or perhaps loss of retractability.

• STRENGTH

For a given foil area, the aspect ratio affects both the applied bending moments and the capability of the foil to resist these moments. Since the effects are opposing, a limit to the aspect ratio is established.

For simplicity consider first a foil with a single strut. If the spanwise load distribution is elliptical, the maximum bending moment in the foil occurs at the strut and is

$$M(o) = \frac{Lb}{3\pi} = 0.106\ Lb^{*}$$

$$= \frac{L}{3\pi}\sqrt{S \times AR}$$

This shows how the bending moment due to lift increases with increasing aspect ratio, for constant lift and area.

The bending strength of the section at midspan determines a maximum allowable bending moment, which is

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*For a uniform spanwise load distribution the moment is

$$M(o) = 0.125\ Lb$$

while a triangular spanwise load distribution gives

$$M(o) = 0.083\ Lb$$

Thus for realistic variations of spanwise load distribution the factor in this equation will not vary greatly.

$$M(o)_{al} = sz_o = sk\, c_o t_o^{\,2} = sk\, c_o^{\,3}\, \tau^2$$

where: s is the allowable bending stress

z_o is the section modulus

c_o is the chord at midspan

t_o is the thickness at midspan

$\tau = \frac{t_o}{c_o}$ is the thickness ratio

$k = \frac{z_o}{c_o t_o^{\,2}}$ is a coefficient depending on the section shape and on the skin thickness

For a foil which is tapered the chord at midspan is

$$c_o = \frac{2}{1+\lambda} \cdot c_m$$

$$= \frac{2}{1+\lambda} \sqrt{\frac{S}{AR}}$$

where: $\lambda = \frac{c_t}{c_o}$ is the taper ratio

c_t is the tip chord

$c_m = \frac{c_t + c_o}{2} = \frac{S}{b}$ is the mean chord

Then $M(o)_{al} = sk\tau^2 \left(\frac{2}{1+\lambda}\right)^3 \left(\frac{S}{AR}\right)^{3/2}$

Upon equating the applied bending moment to the allowable moment, the following relation is obtained:

$$\frac{L}{3\pi} \sqrt{S \times AR} = sk\tau^2 \left(\frac{2}{1+\lambda}\right)^3 \left(\frac{S}{AR}\right)^{3/2}$$

Thus the maximum permissible aspect ratio is:

$$AR_{MAX} = \tau\left(\frac{2}{1+\lambda}\right)^{3/2} \sqrt{\frac{3\pi skS}{L}}$$

which shows the effect upon the allowable aspect ratio of varying the thickness ratio, the taper, the area of the foil and the allowable stress and the solidity of the structure.

If two or more struts are used with a single foil, a different relation will be obtained, depending on the strut spacing and on whether the strut spacing is varied when the aspect ratio is varied. Since so many uncertainties are involved it will be necessary to treat each such case on its own merits. The essential dependence on area, thickness ratio and allowable stress should remain, nevertheless.

III.2.1.3.b Sweepback

The sweepback, or more simply the sweep, is defined by the angle, ω, between a straight reference line in the foil and the transverse direction. The reference line, or sweep line, is usually the locus along the span of points at some constant fraction of the chord lengths. For straight tapered planforms the quarter-chord line is often used. The sweep is taken as positive if the tip is aft of the center of the foil.

Sweepback of the leading edge is considered to be desirable to promote the shedding of seaweed and other trash from the foil. It is not at all clear, however, how much leading edge sweep is necessary for this purpose. A certain amount of sweep will result quite naturally from the use of taper. It does not appear possible to present quantitative guidelines at this time.

The hydrodynamic and structural consequences of sweepback are most important criteria for the selection of this characteristic of the planform. Very briefly, the introduction of sweepback can increase the speed for cavitation-free operation with a given section at a given lift coefficient. Alternatively a thicker section may be used and an increase in strength achieved. On the other hand, sweepback alters the spanwise load distribution and, hence, the induced drag. In this respect the effects of sweep and taper are inextricably correlated so that an optimum combination must be sought. These effects are discussed in more detail in the following paragraphs.

. SWEPT FOIL OF INFINITE ASPECT RATIO

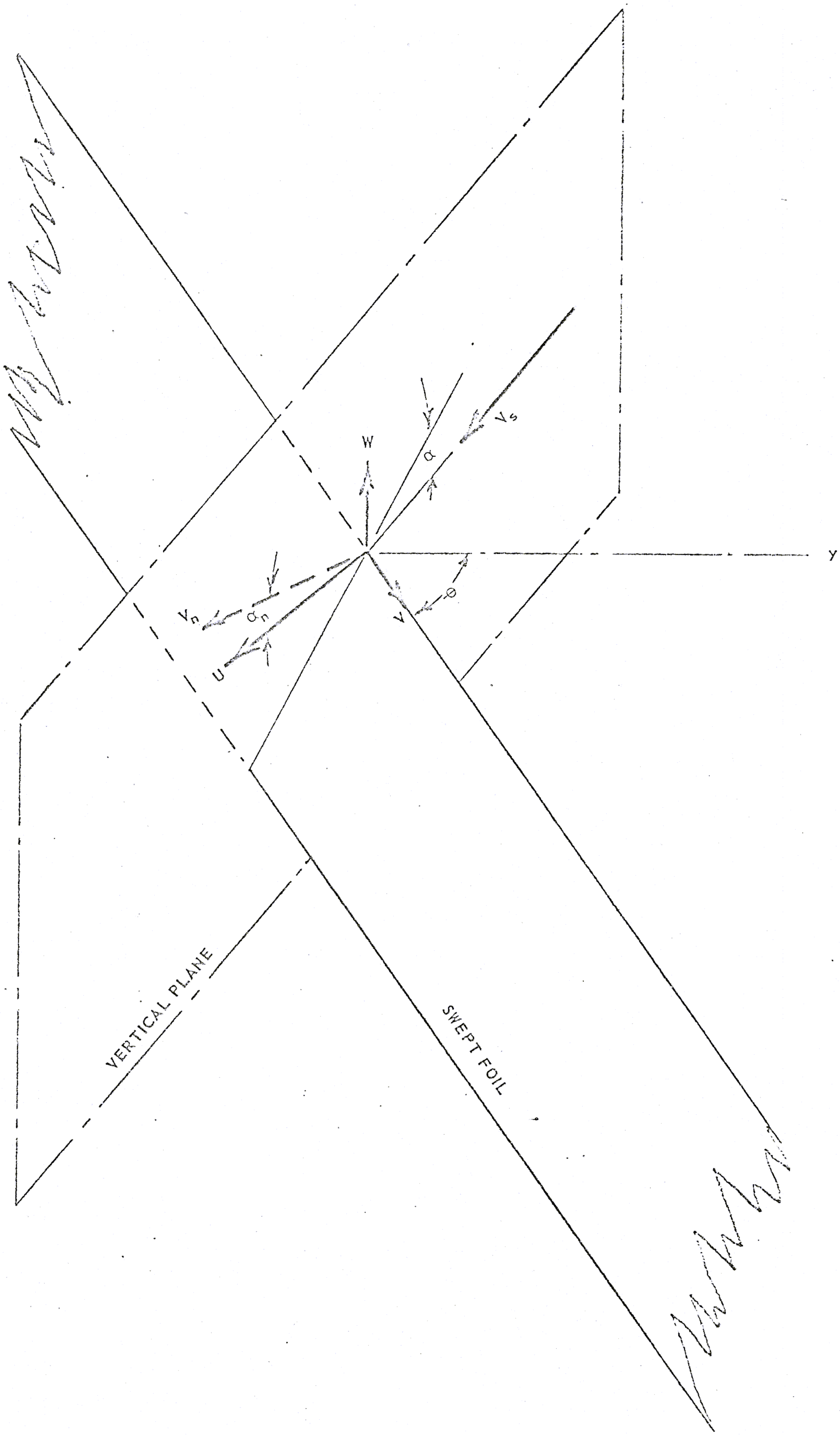
In order to exhibit the essential effects of sweep on the slope of the lift curve and on cavitation, without involving the complications of taper and of tip and mid-span discontinuities, it is useful to consider a foil of infinite aspect ratio. Let the foil, at zero angle of attack, be rotated in its plane to the sweep angle, φ . Now let the incidence of the foil be increased by rotation about the y-axis through the angle, α . The stream velocity, V_s , can then be resolved into three orthogonal components; see Figure III.2-3:

1. $W = V_s \sin \alpha$, normal to the plane of the foil.
2. $V = V_s \cos \alpha \sin \varphi$, parallel to the sweep line.
3. $U = V_s \cos \alpha \cos \varphi$, in the plane of the foil and normal to the sweep line.

For the inviscid approximation to the real flow, assuming only that the Kutta-Joukowski condition is satisfied at the trailing edge, the total flow can be obtained by superposition* of the flow with stream velocity V (parallel to the sweep line) and a flow with stream velocity V_n (normal to the sweep line). But the velocity in the first of these component flows is uniform throughout the field and is everywhere normal to the velocity in the second, which is confined to planes normal to the sweep line. Consequently the flow along the sweep line has no effect on the pressure, which is determined entirely by the normal flow.

The magnitude of the component of the stream in a plane normal to the sweep line is

*Superposition is justified since the inviscid flow has a potential which satisfies the linear Laplace equation.



RESOLUTION OF VELOCITIES OVER A SWEPT FOIL OF INFINITE ASPECT RATIO

FIGURE III.2-3

$$V_n = \sqrt{U^2 + W^2}$$

$$= V_s \sqrt{\sin^2 \alpha + \cos^2 \alpha \cos^2 \varphi}$$

$$= V_s \sqrt{\cos^2 \varphi + \sin^2 \alpha \sin^2 \varphi}$$

$$\doteq V_s \cos \varphi$$

The velocity component V_n forms an angle, α_n, with the plane of the foil such that

$$\tan \alpha_n = \frac{W}{U} = \frac{\sin \alpha}{\cos \alpha \cos \varphi} = \tan \alpha \sec \varphi$$

or, approximately

$$\alpha_n = \alpha \sec \varphi$$

The increment of lift for a unit area of the foil, due to angle of attack, is

$$\frac{\Delta L}{S} = \frac{1}{2} \rho a_o \alpha_n V_n^2$$

where $a_o \doteq 2\pi$ is the two dimensional lift curve slope.

But this may also be expressed as

$$\frac{\Delta L}{S} = \frac{1}{2} \rho a_\varphi \alpha V_s^2$$

where a_φ is the effective lift curve slope for the swept foil.

Thus $$a_\varphi = a_o \frac{\alpha_n V_n^2}{\alpha V_s^2}$$

$$= a_o \cos \varphi$$

which implies a reduced sensitivity of a swept foil to angle of attack changes due to wave action.

Now suppose that the section of the foil has specially favorable chordwise pressure distribution at the ideal angle of attack, α_i, when the lift coefficient is C_{L_i}. In order to preserve this pressure distribution when the foil is swept, make $\alpha_n = \alpha_i$. If the lift coefficient is now referred to the stream velocity, it will be

$$C_L = C_{L_i} \frac{V_n^2}{V_s^2} = C_{L_i} \cos^2 \varphi$$

The minimum pressure coefficient will be similarly affected. Thus if the critical cavitation index is $\left.\sigma_i\right)_o$ for the section, at C_{L_i}, it will be

$$\left.\sigma_i\right)_\varphi = \left.\sigma_i\right)_o \cos^2 \varphi$$

when the foil is swept. Thus a section which will be on the verge of cavitating at a speed $\left.V_c\right)_o$ when unswept may be operated at a at a speed $V_s = \left.V_c\right)_o \sec \varphi$ on a swept foil, with the lift coefficient reduced by a factor of $\cos^2 \varphi$.

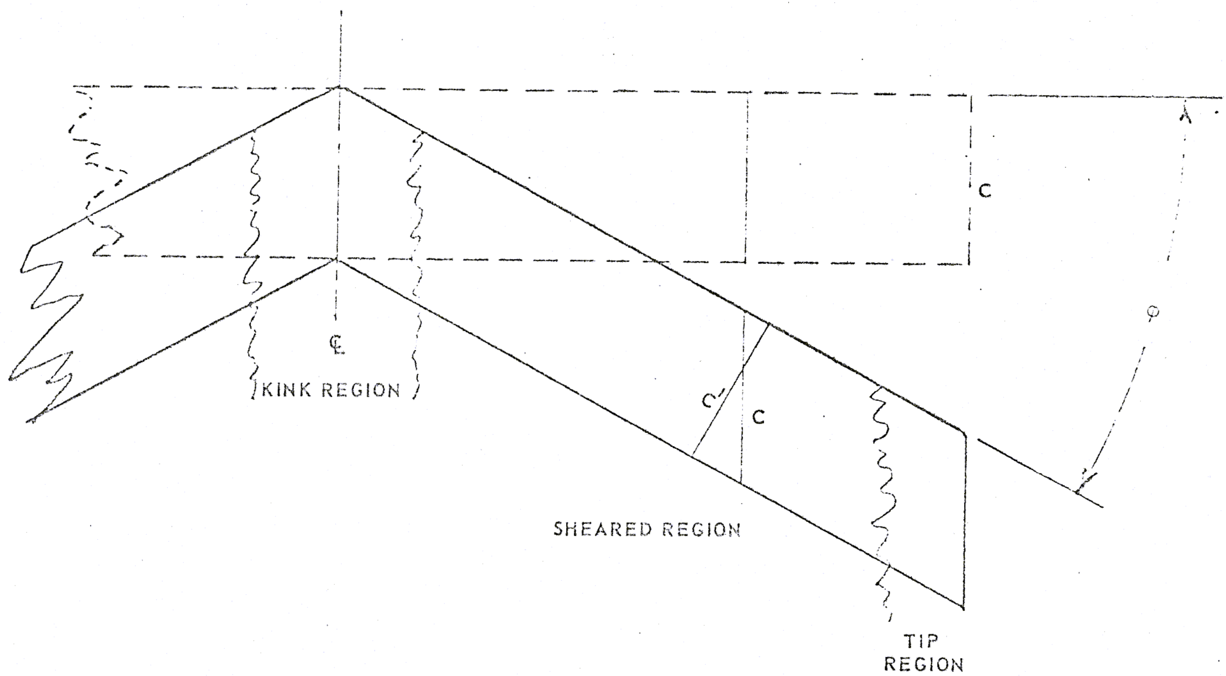
In view of the effects of sweep on the lift coefficient and the cavitation inception speed it is appropriate to compare the foil performance with that of an unswept foil designed to operate at the same lift coefficient and the same cavitation index. On Figure III.2-18, showing the lift/cavitation characteristics of the NACA 66 Sections, the operating points for constant lift coefficient (referred to the stream velocity) at various sweep angles will lie on the same radius vector from the origin. If we choose an unswept foil to provide a lift coefficient of 0.2 at a cavitation index of 0.4, a thickness ratio of 0.087 may be used. The width of the cavitation envelope will be about 2.2°.

A foil swept back 35° may have a section designed to provide a lift coefficient of 0.3 at a cavitation index of 0.6. It may accordingly have a thickness ratio of 0.125 and the width of the cavitation envelope will be 4°, corresponding to an angle of attack range of $4^\circ \times \cos 35^\circ = 3.3^\circ$ in the streamwise direction. The increased allowable thickness ratio and the increase in width of the cavitation envelope are the most important hydrodynamic advantages of sweepback. The implications with respect to real foil design will be discussed in the following section.

.. FINITE SWEPT FOIL

For a real foil of finite aspect ratio the performance is dependent on the total geometry of the planform, including aspect ratio and taper, as well as twist and spanwise variations of section geometry. We consider for the moment the effects of sweep alone and defer until later a discussion of combined effects.

In order to isolate the effects of sweep from those of other



A SWEPT-BACK FOIL OF FINITE ASPECT RATIO

FIGURE III.2-4

characteristics, compare a rectangular foil with a swept foil whose planform is obtained by shearing that of the unswept foil. The area and the span will be the same, hence also the aspect ratio. Thus at the same overall lift coefficient both the induced drag and the profile drag will be the same (except for secondary effects which will be discussed). The length of the foil as a beam will, however, be increased by the factor sec φ and the bending moments correspondingly increased.

Thwaites* has shown that, in the vicinity of the "kink" at midspan, the spanwise loading on the foil with sweepback is reduced

*Reference 1, pp. 273, 320.

while, near the tips, the loading is increased as compared with an unswept foil. If the aspect ratio is not too small, there will be a substantial part of the semi-span where the flow and the consequent loading will be similar to that on a swept foil of infinite aspect ratio. This is usually termed the sheared region, but it must not be supposed that there are sharp lines of demarcation between the three areas described.

.. The Sheared Region

For those sections at least 1/2 chord length distant from the center, and as far from the tip of the foil, the pressure distribution, like that on the swept foil of infinite aspect ratio, will be dominated by the component of flow normal to the sweep line. In carrying out the design of a swept foil of sufficiently large aspect ratio it is, therefore, preferable to specify the geometry of sections normal to the sweep line. The effective chord length is then $c' = c \cos \omega$. The section thickness may be selected from Figure III.2-18 for a cavitation number $\sigma_o = \frac{\sigma_\omega}{\cos^2 \omega}$ and for a lift coefficient $C_L)_o = \frac{C_L)_\omega}{\cos^2 \omega}$, where σ_ω and $C_L)_\omega$ are the cavitation index and the required lift coefficient based on the craft velocity. It is to be noted that, with finite aspect ratio, there is a downwash associated with the trailing vorticity which reduces the effective angle of attack at each section. In general, the effective angle of attack and the lift coefficient will vary along the span of the foil. The spanwise variation of loading and lift coefficient will be discussed in Section III.2.1.3.c, where the combined effects of sweepback and taper are examined. The lift curve slope for the swept foil of finite aspect ratio is also effected by the taper and will be discussed in the next section.

The allowable thickness fraction will be larger than for an unswept foil and, in spite of the reduction in effective chord length, the potential strength may increase more than the bending moment so that a structural benefit accrues.

.. The Kink Region

The Effect of the kink at midspan can be illustrated with reference to an untwisted foil of high aspect ratio and with symmetrical sections. At zero angle of attack, hence at zero lift, the minimum pressure on the foil occurs nearer the trailing edge with increasing sweep and is somewhat increased as compared with an unswept foil.*

When the angle of attack is increased the chordwise loading is less peaked near the leading edge when the foil is swept.** Thus at any substantial lift coefficient the minimum pressure on the foil is increased and occurs farther aft with increased sweep. The loading is reduced in the kink region, which tends to reduce the lift curve slope. Thwaites shows good agreement between calculated and measured chordwise loading on thin, uncambered wings of 45° sweep with aspect ratios of 3 and above.

These effects will be modified if the foil planform is tapered, as is discussed in the following section. The possibility of correcting for these distortions of the pressure distribution by the incorporation of twist and spanwise variations of camber are considered in Sections III.2.1.4 and III.2.1.5.

It is frequently necessary or desirable to locate a pod or nacelle at the kink in a swept foil. The effects of the kink on the chordwise pressure distribution will then be a factor in determining the best for-and-aft position of the pod so as to avoid cavitation at the juncture.

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*Thwaites, Reference 1, p. 279 shows effects at $\omega = 45^\circ$

**Thwaites, Reference 1, p. 323 shows effects for "thin wing".

.. The Tip Region

On a swept foil with constant chord the spanwise loading is increased near the tips and the suction peak is moved forward. A similar result occurs on a tapered foil with the spanwise extent measured in terms of the tip chord. The possible adverse effect on cavitation is discussed in Section III.2.1.5.b, in relation to the design of the foil section.

III.2.1.3.c Taper

For a foil with a trapezoidal semi-span planform, the taper is defined by the ratio of the chord at the tip to that at midspan. The taper ratio, λ, is then

$$\lambda = \frac{c_t}{c_o} .$$

It is the usual practice with a tapered foil to vary the thickness in the same proportion as the chord so that the thickness-to-chord ratio is a constant along the span. The discussion in this section will refer to such a foil.

The incorporation of taper has an important influence on the spanwise load distribution, hence on the induced drag and the bending moment, and also on the strength of the foil, hence on the allowable aspect ratio. It involves some added fabrication difficulties, and a consequent increase in cost, but this would appear not to be a serious drawback since the foil structure will be a precision assembly in any event, whereas the hydrodynamic and strength returns are appreciable.

. TAPER AND STRENGTH

An important structural advantage in the use of taper is the increased dimensions of the critical root section which it permits. Larger bending moments can be sustained, consequently the span and aspect ratio can be larger. This is shown in the formula on page III-20 which indicates the following variation of aspect ratio

$$\frac{A_\lambda}{A_o} = \left(\frac{2}{1+\lambda}\right)^{3/2}$$

where: A_λ is the aspect ratio for a taper ratio, λ

A_o is the aspect ratio for an untapered foil.

The use of taper also permits a more effective distribution of strength in relation to the applied bending moment. Assuming that the spanwise distribution of the hydrodynamic loading on the foil is elliptical* then, for a foil with a single strut at midspan,

---

* Practical foil designs will not depart from elliptical loading sufficiently to alter the following consideration.

the spanwise variation of bending moment will be as shown by the solid line in Figure III.2-5.

The potential bending strength at any section, for a solid foil, varies directly as the section modulus of the section envelope. For a foil built up of skin, spars and ribs or frames, the same variation will obtain if all sections are structurally similar. Thus, the allowable bending moment is

$$M_{a\ell} = sz = sz_o \cdot \frac{ct^2}{c_o t_o^2}$$

With constant thickness-to-chord ratio this becomes

$$M_{a\ell} = sz_o \frac{c^3}{c_o^3}$$

where: s is the allowable stress

z_o is the section modulus at the center section

c is the chord at any section

c_o is the chord of the center section

The chord, c, can be expressed as

$$\frac{c}{c_o} = 1 - \frac{2y}{b}(1-\lambda)$$

Thus the allowable bending moment is

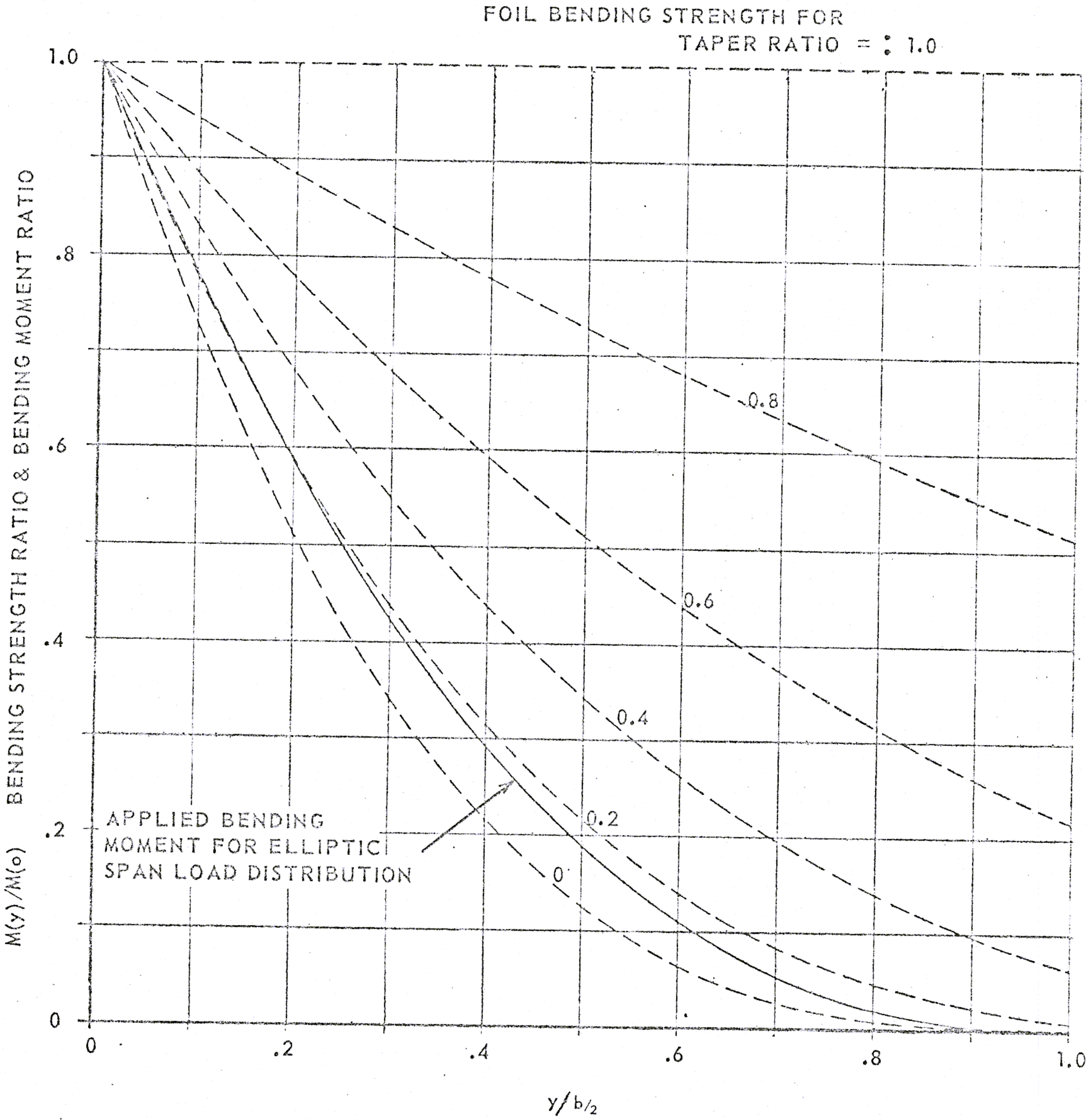
$$M_{a\ell} = \left[1 - \frac{2y}{b}(1-\lambda)\right]^3 M_{a\ell}(o)$$

where: $M_{a\ell}(o)$ is the bending strength at the center

b is the span

y is the distance from the center.

This variation of bending strength is shown by the dotted lines in Figure III.2-5 for several values of the taper ratio. It is clear that, for any taper ratio greater than about 0.2,



EFFECT OF TAPER ON FOIL BENDING STRENGTH

FIGURE III.2-5

there is a substantial excess of strength over that required everywhere outboard of the center. This implies also an excess of weight and cost which can probably be only partly recovered by a more rapid tapering of skin thickness and spar weight than of the chord.

For a foil supported on two struts, similar considerations apply to the outboard panels as to a single-strut foil. The central panel presents a great variety of possible bending moment distributions, depending on the spread of the struts in relation to the span of the foil and on the bending resistance of the struts in relation to the stiffness of the foil. Consequently it does not appear useful to attempt to generalize. Each such foil must be examined individually.

. HYDRODYNAMIC EFFECTS OF TAPER

Taper of the foil planform quite naturally affects the spanwise distribution of the load on the foil, tending to reduce the loading at the tips and increase it at midspan. Since an elliptical spanwise load distribution results in a minimum of the induced drag, one aim in design is to approach such a distribution at least reasonably well.*

For an unswept foil an elliptical spanwise load distribution may be obtained by using an elliptical planform, provided there is no twist and the thickness and camber ratios are constant along the span. In comparison a rectangular planform leads to a load distribution which is more peaked near the tips. A foil with a taper ratio of 0.45 gives a very close approximation to an elliptic load distribution. This is shown in Figure III.2-6(a) which also shows that the section lift coefficient is nearly constant over the whole span.

The use of sweepback tends to shift the loading on the foil out toward the tips, as has been noted in Section III.2.1.3.b. Minimization of the induced drag still requires an elliptic spanwise load distribution but this no longer results from an elliptical distribution of chord lengths. Additional taper is required to achieve a nearly elliptical spanwise load distribution.

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*See NOTE, p.III-35

NOTE:

The elliptical spanwise load distribution provides a minimum of the induced drag of a deeply submerged foil. For a foil near the free surface of the water, the optimum spanwise load distribution is one with the load more concentrated near the tips. The extent of the deviation from the elliptical distribution increases with closer approach to the surface.

This effect is discussed by Nishiyama[10] who shows that the optimum condition requires a spanwise uniform downward induced velocity at the foil. The effect of the free surface is to increase the downwash at midspan compared with the flow when the foil is deeply submerged. To counteract this, a shift of the loading toward the tips is required.

The extent of the modification required is indicated by Nishiyama for a foil of aspect ratio 6.0 at a submergence of $1\frac{1}{2}$ chords at a Froude number of 5 (based on the chord). A procedure is given for calculating the correction for other foils.

Reference to the elliptic loading in the remainder of this section should be understood to imply necessary modification for the free surface effect.

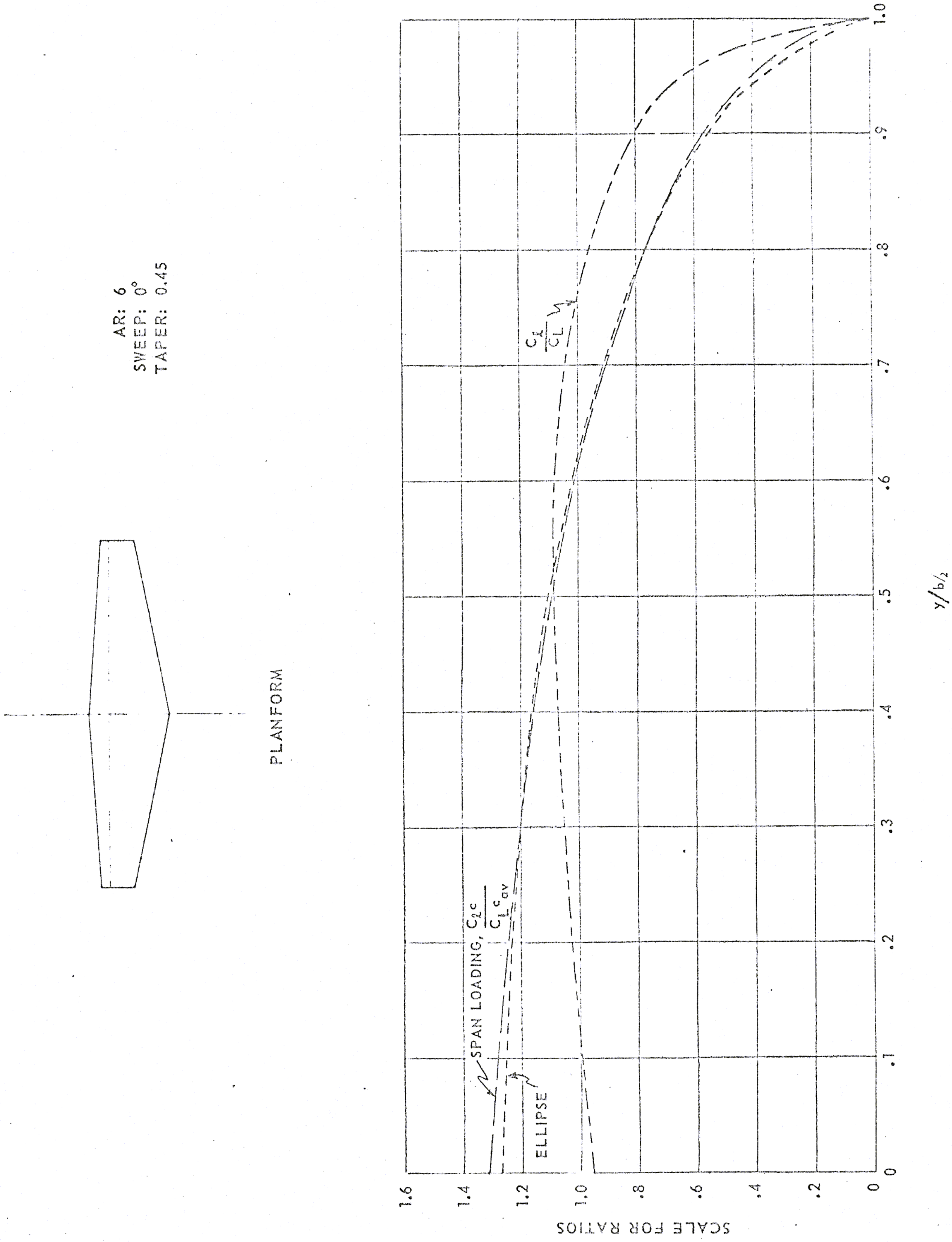

FIGURE III.2-6(a)

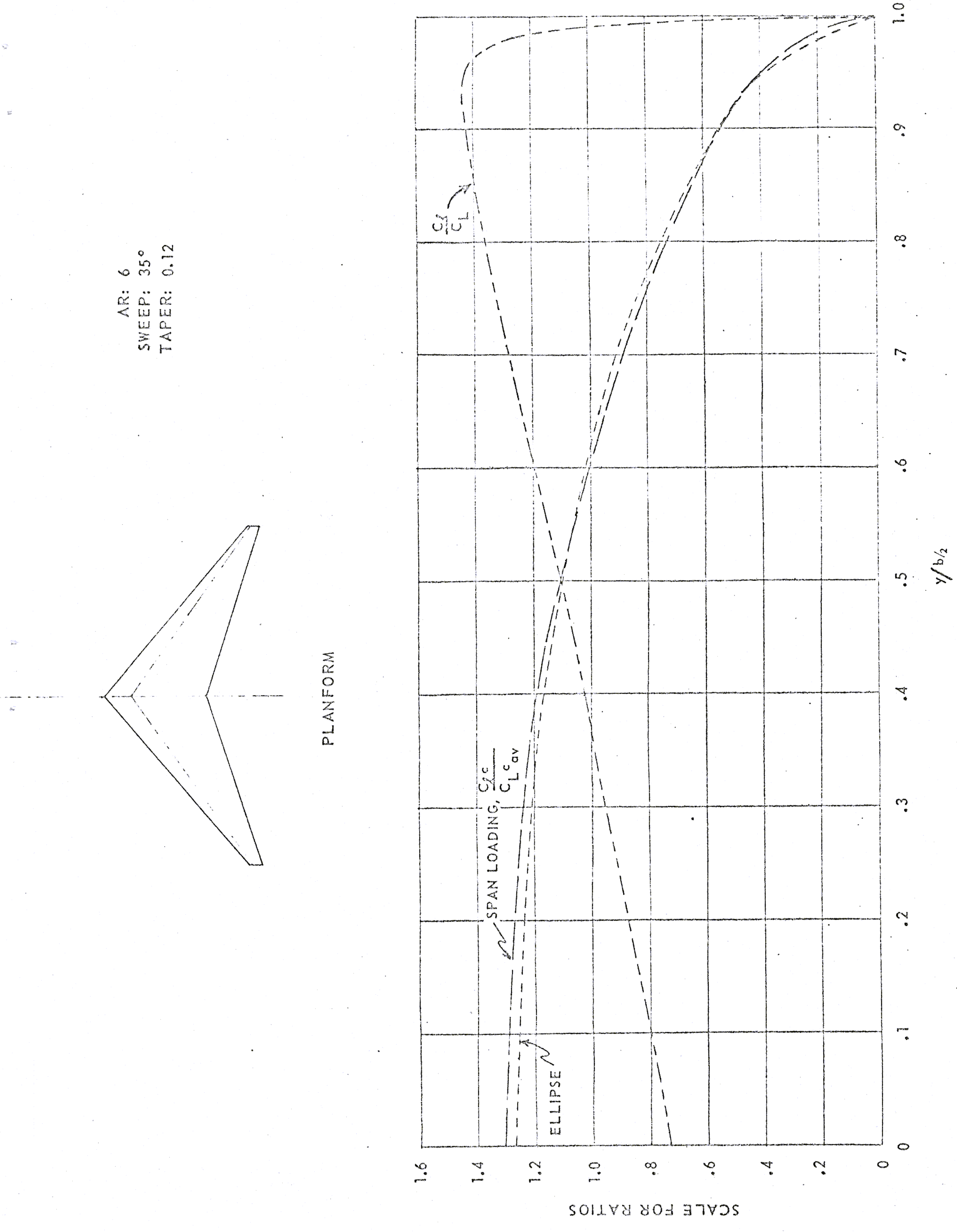

FIGURE III.2-6(b)

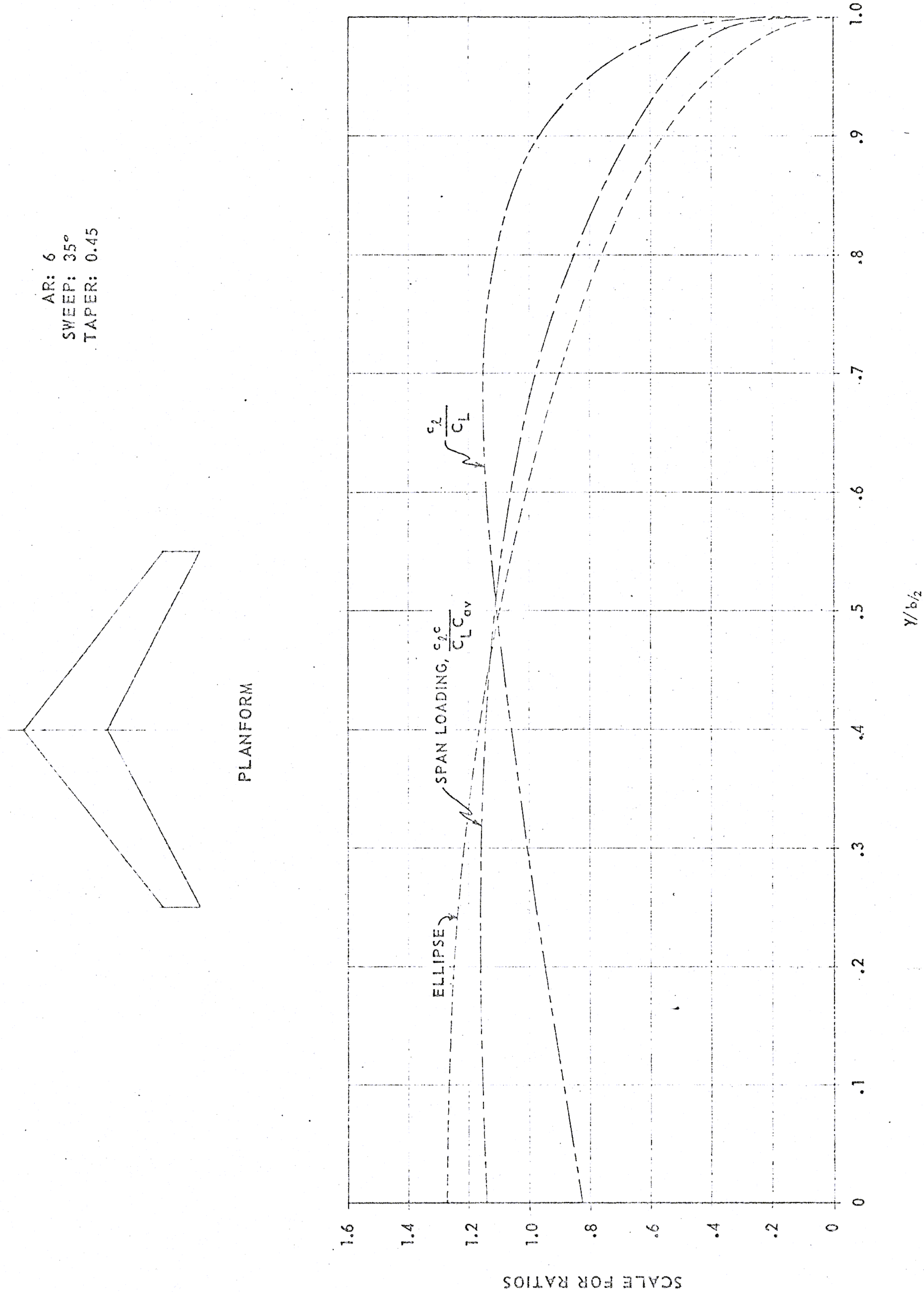

FIGURE III.2-6(c)

An approximate method for calculating the spanwise load distribution on straight tapered foils has been devised by De Young and Harper*, based on a theoretical development by Weissinger[4], leading to the relation shown on Figure III.2-7 between the taper and sweepback for best approximation to elliptic spanwise load distribution. For combinations of sweep and taper not on the line in this plot, the spanwise distribution depends also on the aspect ratio.

The use of taper to improve the spanwise load distribution on a swept foil is not an unmixed blessing, for it leads to a non-uniform spanwise variation of the lift coefficient. This is illustrated in Figure III.2-6(b), which shows the spanwise variation of the load and the section lift coefficient, c_ℓ, for a foil with 35° sweepback and a taper ratio of 0.12. A very good approximation to an elliptical distribution of loading is achieved but the disparity between the planform and an ellipse results in a peak value of the section lift coefficient about 1.46 times the foil lift coefficient. With uniform section geometry along the span the minimum pressure will be lower, at a given foil lift coefficient, than for a foil with uniform section lift coefficient. As a result cavitation will occur at a lower speed. To compensate, increased foil area must be provided with a resultant increase in profile drag.

A spanwise variation of section may be used to partially alleviate the aforementioned difficulty. This is discussed in more detail in Section III.2.1.5. Here we remark that, if an increase in profile drag may be required to maintain the taper necessary to minimize the induced drag, then consideration should be given to accepting some increase of induced drag in order to

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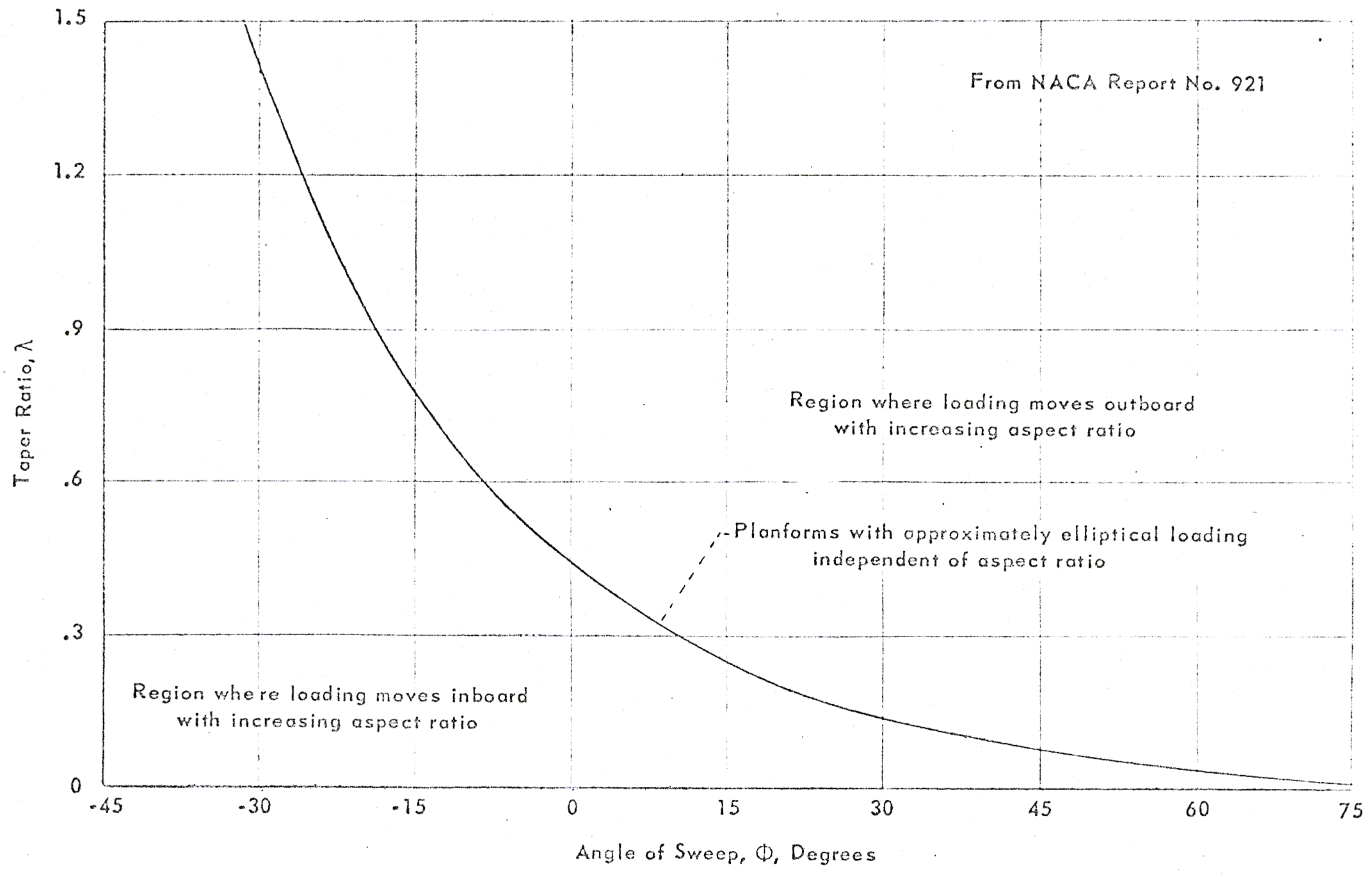
*See Reference 2. Corrections to improve the prediction of the spanwise loading for high aspect ratios are given in Reference 3. Thwaites (Reference 1, p. 348) criticizes the theoretical foundation and indicates a strong preference for the method of Kuchemann[5] which he describes. The method of De Young and Harper has been much used because of the comparative ease of computation.



RELATION OF TAPER RATIO TO SWEEP ANGLE REQUIRED FOR APPROXIMATELY ELLIPTICAL LOADING

FIGURE III.2-7

limit the profile drag while improving the pressure distribution. By way of example Figure III.2-6(c) shows the spanwise load distribution and the variation of section lift coefficient when a taper ratio of 0.45 is used with a quarter-chord sweep angle of 35°. The maximum value of the section lift coefficient is only 1.15 times the foil lift coefficient. The induced drag is 2.7% above that for an elliptical spanwise load distribution. Another advantage of this taper ratio is that, if the foil is twisted to obtain a closer approximation to elliptical span loading, then the lift coefficient variation is also improved simply because a taper ratio of 0.45 provides about the best approximation to an ellipse that can be achieved with straight taper.

The effect of spanwise load distribution on induced drag has been emphasized in the foregoing. It is evident that variations in spanwise load distribution will also affect the root bending moment. Increased taper (lower taper ratio) reduces the bending moment everywhere along the span. The corresponding increase of root bending strength has already been noted. The resulting increase of the allowable span can more than compensate for the effect on the induced drag of the deviation from optimum spanwise load distribution. Consequently an optimum foil may have a taper ratio of 0.20 or less, irrespective of the sweep, with sufficient twist to produce a nearly uniform section lift coefficient. See Section III.2.1.4.a.

Finally recognition should be given to the effect of taper, and of sweepback, on the stalling characteristics of the foil. Since increased taper and sweepback tend to increase the local lift coefficient near the tips, they can lead to the initiation of stall at the tips with an adverse effect on roll control. This is a hazard under conditions of high foil lift coefficient which, for a hydrofoil ship, are to be expected only at takeoff and landing. On the other hand, stalling can only occur if it is possible, through pitch trim and foil incidence change, to achieve a sufficiently large foil angle of attack. This is therefore a problem which must be examined in the design of the control system.

III.2.1.3.d. CURVILINEAR AND IRREGULAR PLANFORMS

The hydrodynamic advantages of the elliptical planform for unswept foils have been noted repeatedly in the foregoing discussion. In spite of this, the straight tapered planform has been emphasized because it appears to be much easier, and therefore cheaper, to fabricate. For smaller craft, however, the use of solid cast or forged foils may be not unattractive.* The techniques of marine propeller blade manufacture are then applicable and the compound curvatures resulting from curvilinear planforms are perhaps less objectionable.

When a foil with elliptical planform is swept back, the loading is shifted outward and the advantage of elliptical spanwise load distribution is lost. To correct the load distribution the planform must be tapered, increasingly with increased sweepback. Thwaites** shows the form required with 45° sweep angle. With this tapering, the local section lift coefficient is increased near the tips, in the same way as with a straight tapered planform.

An attempt to maintain a constant lift coefficient over the whole span of a swept foil, without twist, leads to inverse taper** which exacerbates the problem of providing adequate root strength. A good solution may be to maintain the elliptical planform and to use twist and camber variation to control the spanwise load distribution, and the chordwise loading, as described in Section III.2.1.5.b,iv.

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*The foil area varies approximately as the displacement, Δ, for a given speed. Therefore the volume of the foils, and the weight of a solid foil, varies as $\Delta^{3/2}$.

** Reference 1, Figure VIII.21 and Figure VIII.37

When employing rectilinear, polygonal planforms it may be advantageous to vary the taper stepwise along the span. A better approximation to an elliptical planform may be achieved by increased taper near the tip, for example. On the other hand the strength of the foil at the critical root section can be improved by increased taper near the center. This will also tend to reduce the loading and increase the minimum pressure in a region which is likely to be critical for cavitation because of the presence of a strut and pod. The effects of such modifications on the spanwise load distribution can be roughly estimated by the methods of De Young and Harper[2], provided the sweep line remains straight.

III.2.1.4 TWIST

The twist of the foil is measured by the angle between the section chord line and a reference plane which may be taken through the chord of the center section. Thus the twist is a variable function of the spanwise coordinate. The foil may be built with initial twist to control the spanwise load distribution. Twist may also result from elastic deflection of the foil under load. It is the resultant twist under load with which we are concerned. Thus, in order to achieve the desired span loading. it may be necessary to build in initial twist different from that sought under the designed load.

Because of the tendency of welded, built up structures to warp during fabrication it becomes a serious problem to assure the desired configuration in the completed structure. For this reason the designer need not hesitate to specify twist, just to avoid fabrication difficulty.

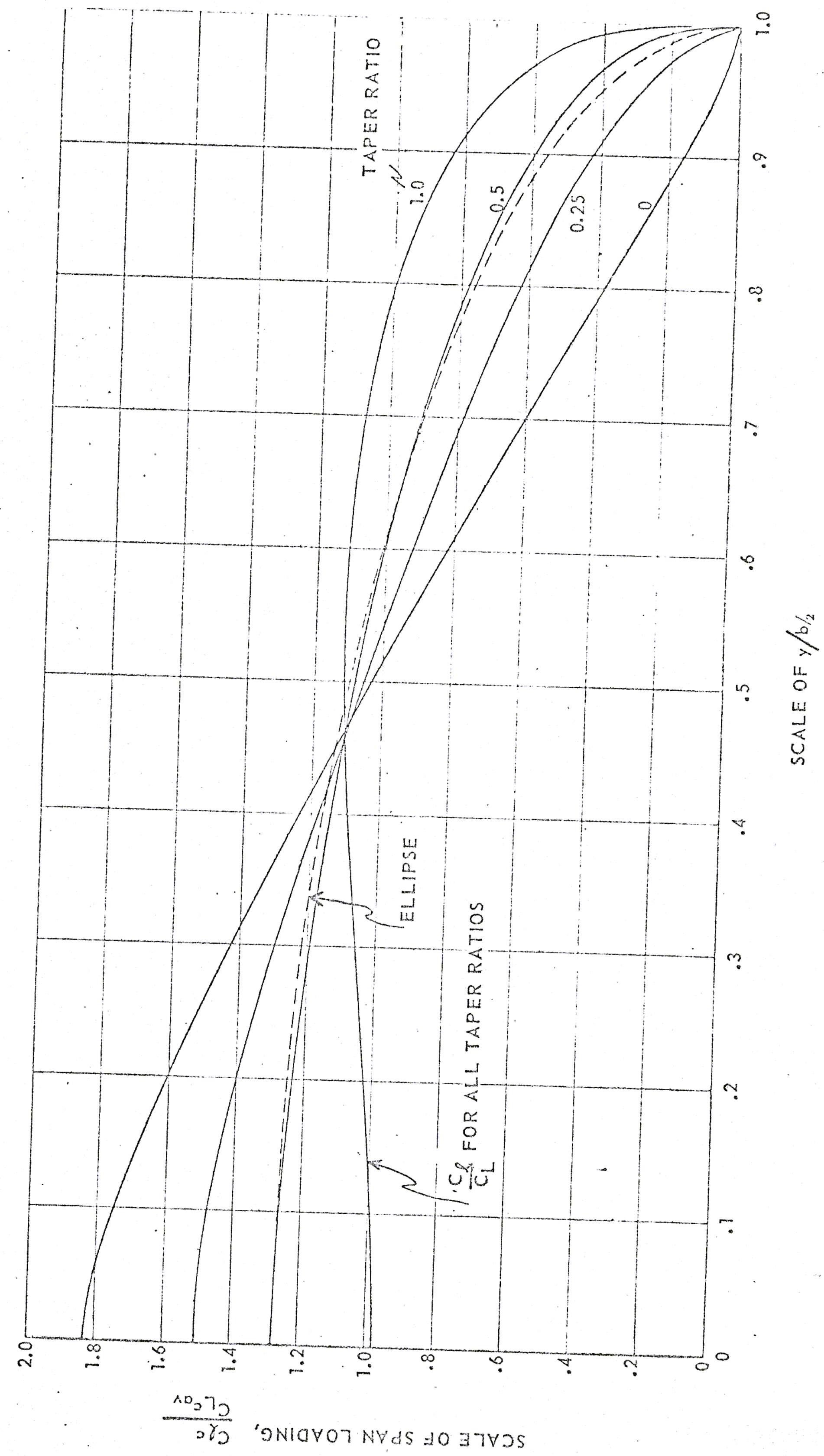
Several factors influence the spanwise loading and must be considered in determining the twist. These are:

a. The planform
b. Proximity to the free surface
c. Downwash from other foils
d. Spanwise variations of camber
e. Elastic deformation

The nature of these influences and the correction obtainable through twist are discussed below.

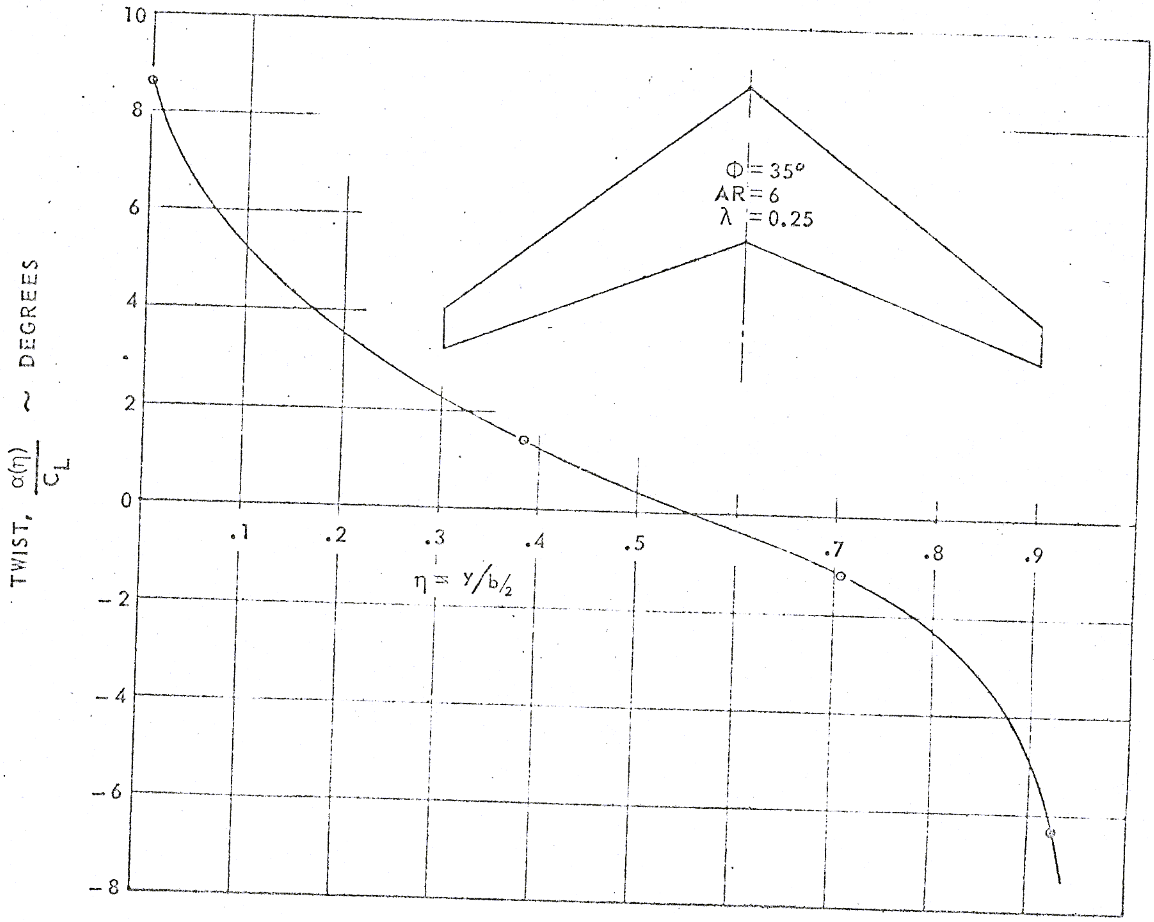
III.2.1.4.a The Twist of an Uncambered Foil in a Free Stream

In the preceding Section III.2.1.3 we have seen how the spanwise load distribution and the local section lift coefficient are affected by taper and sweepback for a flat foil; that is, for a thin foil without twist and with uncambered sections. It was observed that, for an unswept foil, a taper ratio of 0.45 produced a nearly elliptic spanwise load distribution and at the same time a nearly uniform section lift coefficient.



SPANWISE LOAD DISTRIBUTION WITH GIVEN LIFT COEFFICIENT VARIATION, FOR SEVERAL TAPER RATIOS

FIGURE III.2-8



TWIST OF UNCAMBERED FOIL TO GIVE SPAN LOADING OF FIGURE III.2-8

FIGURE III.2-9

When the foil is swept back, increasing taper is required to achieve an elliptical spanwise load distribution. But then the section lift coefficient variation becomes increasingly peaked near the tip of the foil, as shown in Figure III.2-6(b). By twisting the foil it is possible to achieve a desired spanwise load distribution with arbitrary taper and sweepback. As an alternative, to delay cavitation, the foil can be twisted to maintain a nearly uniform local section lift coefficient, again for given taper and sweepback.

It is unrealistic to require a constant lift coefficient all the way to the tip, in view of the inevitable tip leakage flow. And if it could be achieved, the resulting concentration of streamwise vorticity into the tip vortices would result in early tip vortex cavitation. It will usually be desirable, also, to preserve some reduction of the lift coefficient at the center, because of the strut and pod flow interference effects to be expected there. It is suggested tentatively that the spanwise variation of section lift coefficient shown in Figure III.2-6(a), resulting from the use of a 0.45 taper ratio on an unswept foil, may be desirable for any combination of sweep and taper. The corresponding spanwise load distributions, shown on Figure III.2-8 for a range of taper ratios from 0 to 1.0, appear acceptable over a wide range of taper.

The twist required to produce this spanwise variation of lift coefficient and loading has been calculated, by the method of De Young and Harper,[2] for a foil of aspect ratio 6.0 with 35° sweepback and 0.20 tpaer ratio. The result is shown in Figure III.2-9. For this foil the induced drag is increased about 6% above that of a foil of the same aspect ratio with elliptic spanwise load distribution.

When compared with a foil of 0.45 taper ratio which --- with twist appropriate to the sweep --- can provide both an advantageous spanwise load distribution and a nearly constant lift coefficient, the more highly tapered foil can have a 25% larger aspect ratio which results in the induced drag being only 85% as large. Both the increased aspect ratio and the reduction of loading near the tips contribute to a reduction of tip vortex strength. In summary, then, the incorporation of twist can permit the attainment of the benefits of sweepback and taper with a small drag penalty to assure freedom from early local cavitation.

A foil can be twisted to provide desired spanwise loading or lift coefficient variations only at one foil lift coefficient. At any other speed, or total lift, the added lift coefficient is distributed as would be the total lift for an untwisted foil with the same planform. Thus it must first be decided for what speed the twist should be designed, then the off-design-speed effects must be examined. Finally, modification of the planform or section design may be required to achieve freedom from cavitation over the whole range of flying speeds.

III.2.1.4.b Effect of the Free Surface on the Spanwise Loading

The spanwise load distribution on a foil is affected by proximity to the free surface in a way which may be corrected by incorporating an increment of twist. The amount of twist required is proportional to the lift coefficient and inversely related to the submergence, and varies also with the Froude number and the planform. The correction can, therefore, be exact only at one speed and one submergence for a particular foil.

The effect has been discussed, from a theoretical point of view, by Nishiyama[10] who shows that the circulation at midspan (hence also the loading there) is reduced approximately 10% for a rectangular foil of aspect ratio 6.0 at a Froude number of 5 (based on the chord) and a submergence of $1\frac{1}{2}$ chords. Near the tips no reduction occurs. This is consistent with the two-dimensional results of Hough and Moran[14] who show that the loss of lift will be greatest at somewhat lower Froude numbers, depending on the submergence, in agreement with Nishiyama's work for the finite aspect ratio foil.

Nishiyama derives an optimum condition, from the minimum drag point of view, which requires a constant induced vertical velocity along the span. Since the reduction of loading at mid-span, as compared with the distribution at deep submergence, indicates an increase in the induced velocity there due to the surface effect, he proposes to increase the loading near the tips to compensate. Accordingly, for a foil of elliptic planform, he suggests the use of twist to increase the angle of attack toward the tips. The result is a further deviation of the spanwise loading from the elliptic variation, for this foil.

If, on the other hand, emphasis is put on delay of cavitation and a spanwise uniform lift coefficient is sought, then it appears that twist must be applied in the sense to decrease the angle of attack and the loading near the tips. An increase of induced drag will result. Extension of Nishiyama's work will be required to provide a basis for design.

These considerations suggest that, for operation near the surface, a planform which is fuller near the tips than the ellipse will be superior. Put another way, it appears that less taper is indicated when surface effects are appreciable.

III.2.1.4.c Foil Interaction

An isolated foil developing lift creates a disturbance throughout the field of flow. The disturbance is attenuated rapidly with increasing distance away from the foil, except for the effects associated with the surface waves generated and with the streamwise vorticity extending downstream in the wake of the foil. Consequently, for the usual foil configurations with forward and after foils separated by a distance comparable to the span of the larger foil, the most important interference effect is that of the forward foil on the after foil. In addition, if split main foils are used, there is also a mutual influence of each upon the other which can be appreciable only near the adjacent foil tips.

The concern here is the vertical velocity or downwash behind the forward foil which is variable along the span, causing a spanwise variation of the angle of attack of the after foil. Equivalent twist of the after foil can be used to correct for the spanwise load variation which would otherwise result. The pattern of twist required depends, of course, on the downwash pattern and on the relative size and positions of the forward and after foils. Conversely, consideration of interference is an important factor in selection of the foil configuration. In fact avoidance of regions of peak downwash may be the most practical solution, for which an understanding of the downwash pattern is a prerequisite.

. THE HYDROFOIL WAKE

The wake of the hydrofoil bears a fundamental similarity to the frictional wake and downwash field behind an airplane wing. The hydrodynamic field is, however, significantly influenced by the presence of the free surface and is distorted by surface waves generated by the passage of the foil and travelling with the foil. These concepts form the basis of theoretical analyses of the flow from which the velocities at the after foil can be calculated.

The aerodynamic field has been extensively studied, because of its important effects on the tail surfaces of the airplane, and we will summarize the known characteristics in the following section. Extensive theoretical studies of the surface and wave effects have been carried out. The results are presented in the form of complicated integrals which have been evaluated only for a few special cases. It is possible at this time, therefore, to give only a very general picture of the modifications and distortions which are produced.

. THE AERODYNAMIC WAKE

The wake behind a wing, or a deeply submerged hydroil, includes a band of turbulence generated by friction in the flow over the foil. This has only a minor influence on the downwash but produces an undesirable environment for a propeller which may be located aft.

The simplest approximate representation of the downwash in the wake behind a foil is by means of a band, or sheet, of streamwise vorticity extending downstream to infinity with a width equal to the span of the foil. In reality this vortex sheet rolls up along its edges and the vorticity is drawn away from the center to form two distinct vortices about 0.8 of the span apart. See Figure III.2-10. But this distortion occurs rather slowly so that, for a distance down stream of several times the foil span, the simple picture of a flat band is adequate.

Such a sheet of vorticity involves transverse velocities, outward from the center below and inward on the upper side, as well as vertical velocities which are the principal concern here. If the spanwise loading of the foil is elliptical, then the vertical velocity is contant across the width of the band. This velocity reaches the value, far downstream, of

$$\frac{w_{x \to \infty}}{V} = \frac{2\ C_L}{\pi\ AR}$$

where: V is the forward velocity of the foil

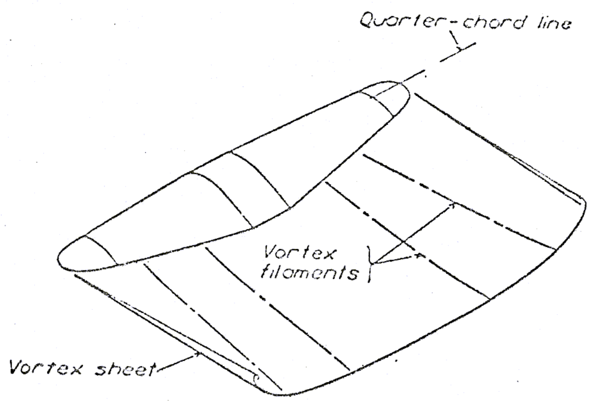
C_L is the lift coefficient

AR is the aspect ratio.

When account is taken of the rolling up of the vortex sheet, the velocity far downstream in the plane of symmetry is reduced, the factor 2 becoming 1.62.

An indication of the magnitude of the downwash can be obtained by relating it to the foil angle of attack, α . Thus in theory, for a foil with elliptic spanwise loading, the lift coefficient is

$$C_L = \frac{2\ \pi\ \alpha}{1 + \frac{2}{AR}}$$



THE VORTEX SHEET BEHIND A FOIL

FIGURE III.2-10

Then the downwash angle on the plane of symmetry far downstream is

$$\epsilon = \frac{w}{V} = \frac{4\alpha}{2 + AR} .$$

Thus, for a foil of aspect ratio 6, $\epsilon = 1/2\ \alpha$. An aft mounted foil set at the same incidence would carry only half the lift it could in a free stream.

Near the foil the velocity induced by the trailing vorticity is reduced, approaching a value at the foil of half the value far downstream. Here, however, it is necessary to include the influence of the bound vortex which represents the circulation around the foil accounting for the lift. With this the downwash in the plane of symmetry becomes approximately*

$$\frac{w(x)}{V} = \frac{2\ C_L}{\pi\ AR} \left[1 + \left(\frac{b}{2\pi x}\right)^2\right]$$

where: b is the span of the foil

x is the distance downstream from the foil

and the approximation is not valid for very small values of x. It is seen that, when the effect of the circulation around the foil is included, the downwash is never less than that far downstream, which is due to the trailing vorticity alone.

Extensive calculations of the downwash in the plane of symmetry close behind a wide variety of wings, both plain and with flaps, have been carried out by Silverstein and Katzoff,[15] who have presented the results in the form of design charts.

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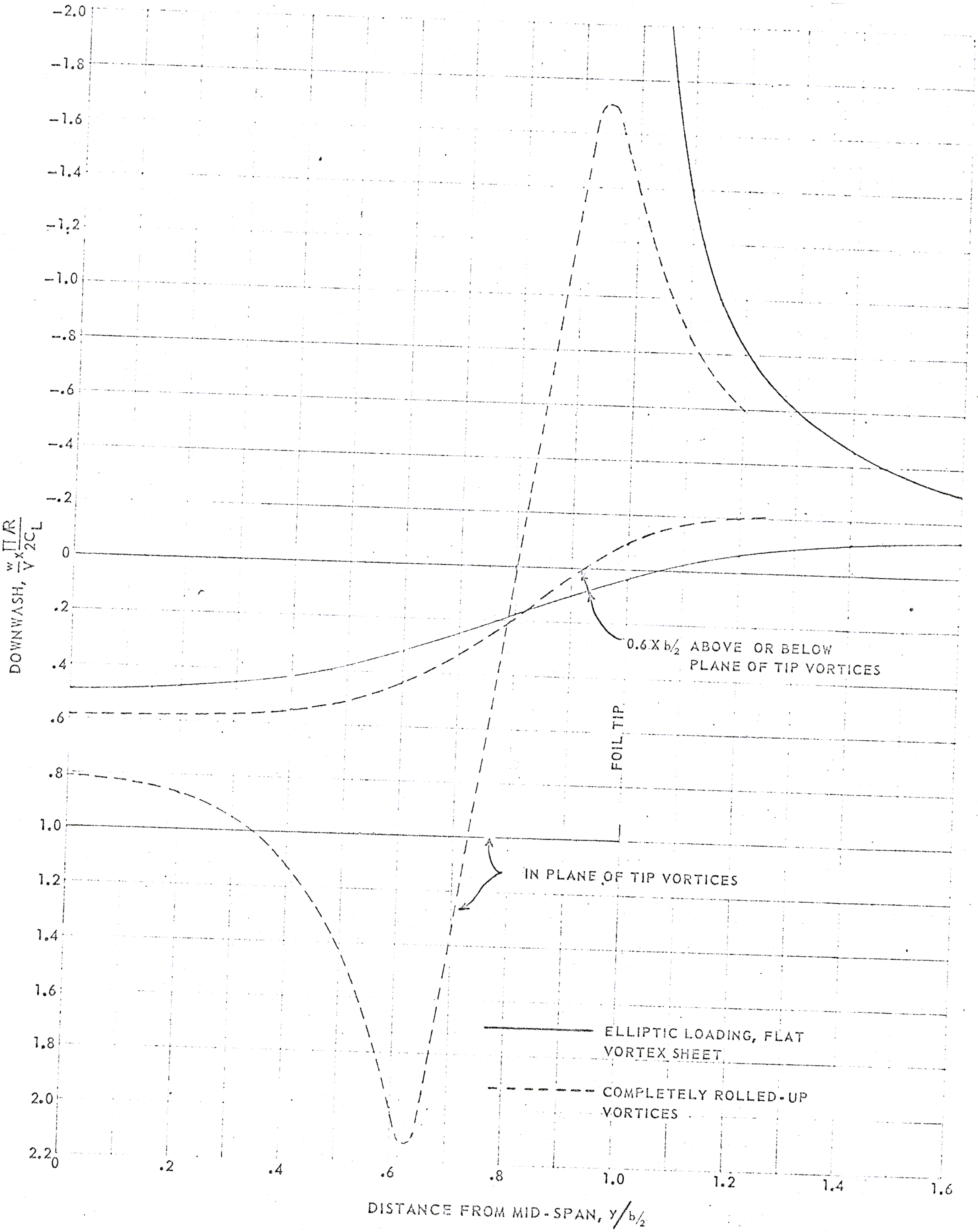
*See Thwaites, Reference 1, p. 545

The spanwise variation of the downwash velocity which strongly influences the spanwise loading on the after foil, changes with increasing distance downstream from the forward foil. Near the forward foil the vortex sheet will be nearly flat and, if the spanwise loading is elliptic, the variation of downwash will be approximately as shown by the solid lines in Figure III.2-11. At a great enough distance downstream so that the vortex sheet is completely rolled up, the spanwise variation of the downwash is as shown by the dashed lines in this figure. As is evident the patterns are significantly different behind the foil in the plane of the tip vortices. At a height of 0.6 of the semi-span above this plane there is very little difference. Outboard of the foil tips, and in the plane of the vortices, there is a strong upwash which persists far downstream and is instrumental in the rolling up of the trailing vortex sheet. The rolling up process, which is illustrated in considerable detail by Spreiter and Sacks,[16] and also by Silverstein, Katzoff and Bullivant,[17] occurs gradually. According to Spreiter and Sacks it can be considered complete at a distance downstream given by

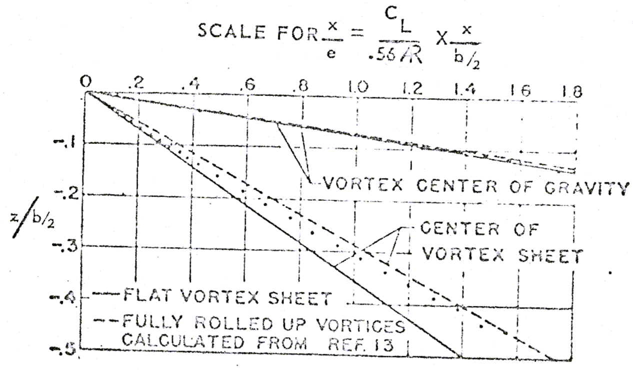
$$e \approx 0.28 \frac{AR}{C_L} b$$

where b is the span. For a typical foil application with an aspect ratio of 6 and a lift coefficient of 0.2, this gives $e \approx 8 b$. Thus at high speed the after foil will be in the region of transition. At lower speeds, particularly for a canard configuration, the vortex sheet from the forward foil may be substantially rolled up at the after foil.



SPANWISE VARIATION OF DOWNWASH BEHIND A FOIL IN A FREE STREAM

FIGURE III.2-11



DOWNWARD DISPLACEMENT OF THE TRAILING VORTEX SHEET FOR ELLIPTIC LOADING

FIGURE III.2-12

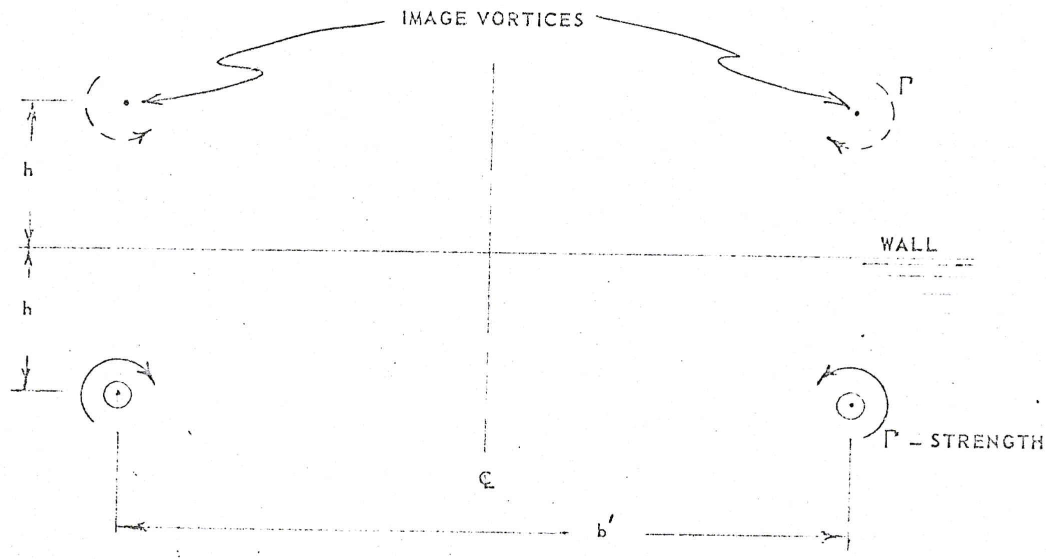
As a result of the downwash, the band of vorticity — and turbulence — in the wake is convected downward at an angle given, in radians, by the ratio $\frac{w}{V}$. The downward displacement of the center of the vortex sheet is shown in Figure III.2-12, measured in semispan units, as a function of the distance downstream. The displacement of the center of gravity of the trailing vorticity is also shown. This of course coincides with the centers of the vortex cores when the sheet is completely rolled up, and indicates the elevation of the maximum downwash velocity in this case. The design charts of Silverstein and Katzoff[15] include curves showing the position of the center of the vortex sheet for a distance downstream of about 3/4 of the span.

From the discussion above it appears that an adequate picture of the downwash behind a deeply submerged foil can be obtained to permit a determination of the need for twist of an after foil. It remains to consider the effects of the free surface and of the surface waves, which will be done in the following.

. EFFECTS OF THE FREE SURFACE ON THE WAKE

Two distinct surface effects operate to modify and distort the aeordynamic wake described in the previous section.

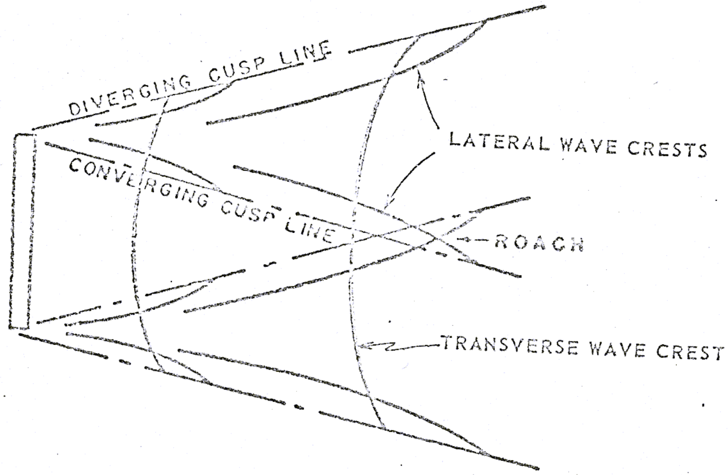
In the first place, considering the trailing vorticity, the



IMAGES OF A VORTEX PAIR IN A RIGID WALL

FIGURE III.2-13

flow will be principally two dimensional in planes perpendicular to the direction of craft motion. The free surface must be both a stream line and a line of constant pressure. It cannot remain plane for, if it were a plane rigid boundary, the pressure would vary spanwise. Consequently the surface will be distorted by streamwise furrows, whose depth and breadth depends on the strength of the trailing vorticity and its submergence. Nevertheless, the flow far downstream will resemble that due to a vortex pair parallel to a rigid wall and certain qualitative aspects of the downwash for this cannonical field, which is pictured in Figure III.2-13, may be carried over to case of the free surface. It is evident that the effect of the wall, which is equivalent to the two symmetrical image vortices, is to reduce the downwash and hence, also, the downward slope of the vortex sheet behind the generating foil and its depth at any point downstream. The trailing vortices will also be drawn together so that the picture of the completely rolled up vortices spaced about 0.8 b apart must be modified.



WAVE PATTERN FROM A HYDROFOIL

FIGURE III.2-14

The second effect of the free surface results from the waves generated by the circulation around the foil and which produce velocities and surface distortions in the wake. The aerodynamic wake will be convected vertically by the waves, subject to the usual law of attenuation of wave orbital velocities with depth, so that the wave velocities will be added to the aerodynamic downwash. The longitudinal component of the wave velocities can be ignored in comparison with the ship speed.

The configuration of the surface waves behind a foil of finite span is pictured in diagrammatic form[18] in Figure III.2-14. Figure III.2-15 shows the complexity of the wave system[19] behind a foil of aspect ratio 20 at several speeds. A small after foil, whose tips did not extend to the converging cusp line, would be subjected to a reasonably predictable wave effect, the vertical velocities being indicated by the streamwise spacing of contours—with appropriate attenuation. An estimate of the effect on spanwise loading and of the indicated corrective twist can be made.

It is evident that, if the span of the after foil is comparable to that of the forward foil, a very complicated spanwise variation of downwash will obtain at the after foil. Split main foils, either forward or aft, will lead to different but equally complex patterns. The twist which might be indicated has not been examined at this time.

In addition to the experimental work done, which is exemplified by Figure III.2-15, two theoretical analysis of the hydrofoil wave system should be noted. The first, by Kaplan, Breslin and Jacobs[20] presents integral expressions for the velocity potential of the flow field, including the free surface effects. The downwash velocity has been averaged over the span of an after foil, and the result is compared with model tests. The indicated agreement is good, but no indication is given of the spanwise variation.

The second study is the culmination of extensive work on hydrofoil wave theory by Nishiyama.[10] Integral expressions for the downwash field are given. A partial reduction is carried out for the downwash velocities on the plane of symmetry. A numerical integration still remains to be accomplished in this case. Thus it appears that very extensive computations are required to apply the available theory to determine the spanwise variation of the downwash as modified by free surface effects. No experimental data are known by which the results of such computations could be tested.

It is suggested that model tests should be carried out, and the results compared with theoretical calculations, to assess the importance of the free surface effects on the spanwise load distribution and cavitation inception speed for a hydrofoil running in the wake of another foil.

. INTERACTION BETWEEN FOILS ABREAST

As shown in Figure III.2-11 there is a strong upwash outboard of the trailing vorticity behind a wing or foil. These curves are for vorticity extending to infinity in both directions, thus they are applicable only at an appreciable distance abaft the generating foil. The theory from which they are derived, that is the Biot-Savart law of induction, indicates that the induced velocities in a transverse plane at the foil will be just one half the values shown. Furthermore, the vorticity just abaft that plane will be distributed spanwise so that the velocity distribution indicated by the solid lines is applicable.

It is clear, then, that the adjacent tips of two foils abreast will be affected by a mutual upwash whose magnitude decreases quite rapidly with increasing spanwise separation. With a tip separation of 18% of the span of either foil, the maximum upwash will be 1/2 the (constant) self induced downwash and will decay toward midspan. The necessary corrective twist can be readily calculated from these curves, for a deeply submerged foil.

When the foils approach the surface there will be a more rapid decrease of the upwash with increasing distance away from the foil tip. The reason for this can be seen from an examination of Figure III.2-13; actual evaluation will require computations based on the theoretical development of Kaplan, Breslin and Jacobs[20] or of Nishiyama.[10]

III.2.1.4.d <u>Effect of Camber</u>

By the calculation described above the incidence of every section of the foil is determined, to give the desired spanwise loading, provided the sections are thin and uncambered. The results can be used to deduce the local flow direction for zero lift at each section.

For a cambered section, however, the local flow direction for zero section lift is not along the chord line in general. There is instead an (effective) angle of attack for zero lift, $\alpha_{L=0}$, which depends on the shape of the camber lines.* Therefore, in order to determine the local incidence of the chord line of a cambered section, the angle of attack for zero lift must be added algebraically to the calculated twist. If the camber is constant along the span, the twist is not affected. But if the

*Abbott and von Doehoff, Reference 12, p. 73.

camber or mean line shape are varied spanwise, as they may well be to control the chordwise loading near the tips or near the kink of a swept wing, then the design twist will be altered.

On a swept foil, in the kink region and near the tips, the angle of zero lift and the lift curve slope are affected and the calculation of the twist is more complex. This problem is treated in detail by Kuchemann[5] and by Brebner,[7] who present computational procedures for determing the twist.

III.2.1.5 SECTION

The shape of the cross section* of the foil has a profound effect on its performance, especially with respect to the strength, the cavitation inception speed and the minimum flying speed. Because the requirements differ, from each of these points of view, the design necessarily involves a compromise.

For a foil with straight tapered planform, the fabrication will surely be easiest if the foil is tapered in thickness in the same ratio and all sections are geometrically similar. In spite of this it may be well worth while to vary the section shape along the span. Almost certainly the skin thickness will be varied, as well as the scantlings of the ribs and spars, so that the construction will not be simple in any event and the advantage of section similarity may be illusory.

III.2.1.5.a Strength

Because of the usually elongated shape of the planform, its method of support, and its loading as a beam, the primary concern is with the bending strength of the foil. The considerations noted in Section III.2.1.3.c indicate that the root section of a single strut foil will almost certainly be critical. For a two strut foil, if the bending stiffness of the struts is appreciable, an analysis of the strut and foil as a redundant frame may be required.

For any given type of foil structure, whether solid or built up, and with a given material, the bending strength is proportional

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*For a foil with appreciable sweepback the pertinent section, from the hydrodynamic standpoint, is that cut by a plane normal to the sweep line. If the foil is also tapered in plan and in thickness, the sections cut by planes parallel to the plane of symmetry are distorted in thickness ratio and also in the chordwise distribution of thickness. It will be simplest to delineate such a foil by sections normal to the sweep line.

to t^2c where:

t is the thickness of the section and

c is the chord.

There is, therefore, a strong incentive to use the thickest possible sections. The chordwise distribution of section thickness will of course have some influence, but the practical variations are small.

The foil will also be subjected to torsion due to chordwise movement of the loading when the lift coefficient has to be varied as a result of speed variations. Elastic twist of the foil also affects the hydrodynamic torsional moments and the possibility of divergence and flutter must be examined. Hence, not only strength but also stiffness is of concern. From these considerations also the maximum permissible thickness will be sought.

III.2.1.5.b Cavitation

Because of the damaging effects of prolonged cavitation, it will be required that the foils be substantially free of cavitation from the minimum flying speed to the maximum speed at which extensive operation is required. Allowance must be made for the effects of waves on the angle of attack and the lift of the foils in rough water. It is necessary, then, to consider carefully the design of foils to avoid cavitation.

The design of the sections of a foil of finite aspect ratio is usually based on the proposition that, at any local section lift coefficient, the chordwise distribution of velocity and pressure is the same as it would be on a foil of infinite aspect ratio at the same lift coefficient. Accordingly, two-dimensional, infinite aspect ratio flow theory and experimental results regarding velocity and pressure distribution in relation to section geometry are applied directly. In particular a wealth of aeronautical data are available.

The validity of the two-dimensional chordwise loading has been sufficiently well established for the design of straight foils. A tendency for the loading to be somewhat shifted toward the leading edge near the tip, as shown by Cohen[6], is compensated for by a simultaneous reduction of the span loading in the tip region. A suitable combination of taper and twist to provide an advantageous spanwise variation of lift coefficient, as discussed in Section III.2.1.4.a, will permit the section of an unswept foil to be designed on the basis of the two-dimensional pressure distribution data described in the following Section.

When a flat foil is swept back, the resulting distortion of the spanwise load distribution shown in Figure III.2-6(c) is accompanied by a chordwise redistribution of load in the kink and tip regions. Nevertheless it will be found that, when twist is used to recover a desirable spanwise load distribution, then a suitably cambered section will provide an acceptable chordwise load distribution at all sections. This is discussed in more detail in Section III.2.1.5.b.iv. First, however, the important characteristics of the two-dimensional chordwise pressure distribution will be examined.

III.2.1.5.b.i <u>Two-Dimensional Pressure Distribution</u>

The essential problem of foil section design for avoidance of cavitation is to control the chordwise pressure distribution so as to limit the peak pressure reduction --- or suction --- at any point on the surface. This is accomplished by suitable shaping of the mean line, or camber curve, in conjunction with the chordwise distribution of the thickness.

The pressure reduction, for a given foil at a given angle of attack, will vary with the square of the speed. Hence it is convenient to express it by means of the nondimensional pressure coefficient, C_P, which is the ratio of the pressure change from that in the free stream to the stagnation pressures. Thus

$$C_P = \frac{p-p_o}{\frac{1}{2}\rho V^2}$$

where p is the pressure at a particular point on the section

p_o is the pressure far ahead of the foil at the same submergence

ρ is the mass density of the fluid

V is the speed

The pressure coefficient is independent of the speed but depends on the shape of the section and the angle of attack and varies along the periphery. Figure III.2-16 shows this variation along both the top and bottom of the NACA 66-(.442)10 section. The upper part of this figure is for an angle of attack of 0.68° at which the section is designed to provide a nearly uniform pressure reduction along the top for about 60% of the chord. The lower diagram shows the variation of the pressure coefficient at an angle of attack of 2.68° and indicates clearly the increased pressure reduction at this off-design condition of operation. It is noteworthy as well that the minimum pressure occurs at different locations on the section profile at these different angles of attack.

Cavitation is expected to occur when the pressure at any point on the foil is reduced approximately to the vapor pressure of water, p_v. But the conditions under which this will occur depend on the submergence, which governs the free stream pressure, p_o. Thus the pressure reduction on the foil must not exceed $p_o - p_v$, which determines the cavitation index

$$\sigma = \frac{p_o - p_v}{\frac{1}{2}\rho V^2}$$

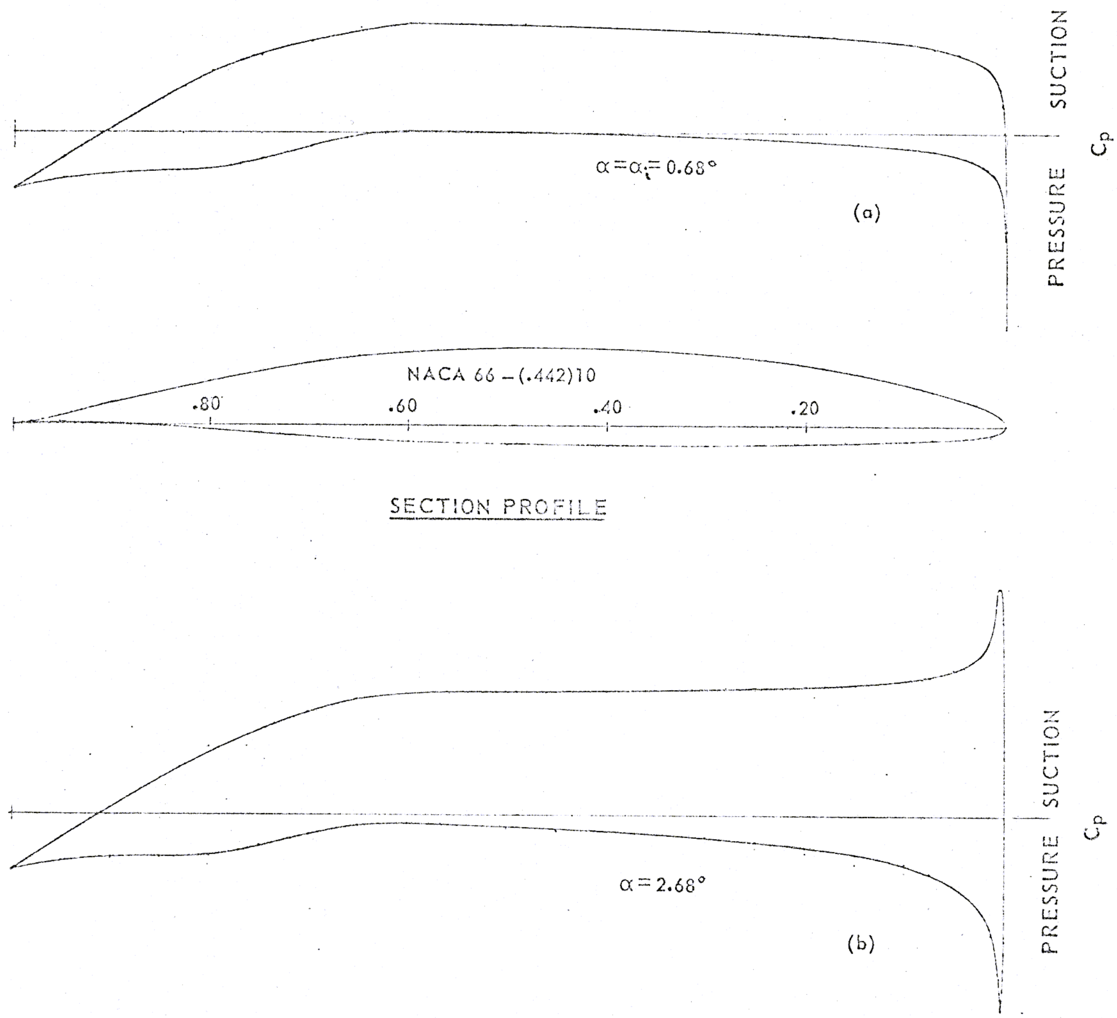
with $p_o = p_a + \gamma h$

where p_a is the atmospheric pressure

h is the submergence

γ is the unit weight of water

To avoid cavitation on a given section at a given angle of attack, the speed and the submergence must be such that $\sigma > -(C_P)_{MIN}$. For given operating conditions, on the other hand, the foil must be designed or chosen so that $-(C_P)_{MIN} < \sigma$ at the required speed, submergence and lift coefficient.



EFFECT OF INCREASED ANGLE OF ATTACK ON THE CHORDWISE PRESSURE DISTRIBUTION

FIGURE III.2-16

A very careful theoretical calculation of the pressure distribution on three families of foil sections has been carried out by Terry Brockett.[11] The results are presented as envelopes showing the minimum pressure coefficient for sections of given thickness and camber as a function of the angle of attack. One of Brockett's diagrams, for the DTMB modified NACA 66 section with maximum camber ratio of 0.03, is shown in Figure III.2-17. For this camber line and amount of camber the ideal angle of attack* is 0.68°. The corresponding lift coefficient is 0.46 for zero thickness and decreases slowly with increasing thickness.

Fixing attention for the moment on the envelope for a foil of thickness ratio $\tau = 0.10$, the minimum pressure coefficient at the ideal angle of attack is approximately 0.51, corresponding to the pressure distribution shown in Figure III.2-16(a). The minimum pressure occurs on the top at about 60% of the chord from the leading edge. As the angle of attack is increased, the minimum pressure is reduced slowly ($-C_p$ increases) corresponding to the nearly vertical part of the envelope. At the same time the point of minimum pressure moves gradually forward along the top of the section. When the angle of attack reaches about 2°, the point of minimum pressure is near the leading edge and further increase in the angle of attack results in a much more rapid pressure reduction. Figure III.2-16(b) shows the concentrated peak of suction very close to the leading edge at an angle of attack of 2.68°.

If the angle of attack is decreased, on this section of 10% thickness, the minimum pressure on the back increases slowly and moves aftward. A more important change in the flow occurs at the nose, however. The stagnation point corresponding to the peak

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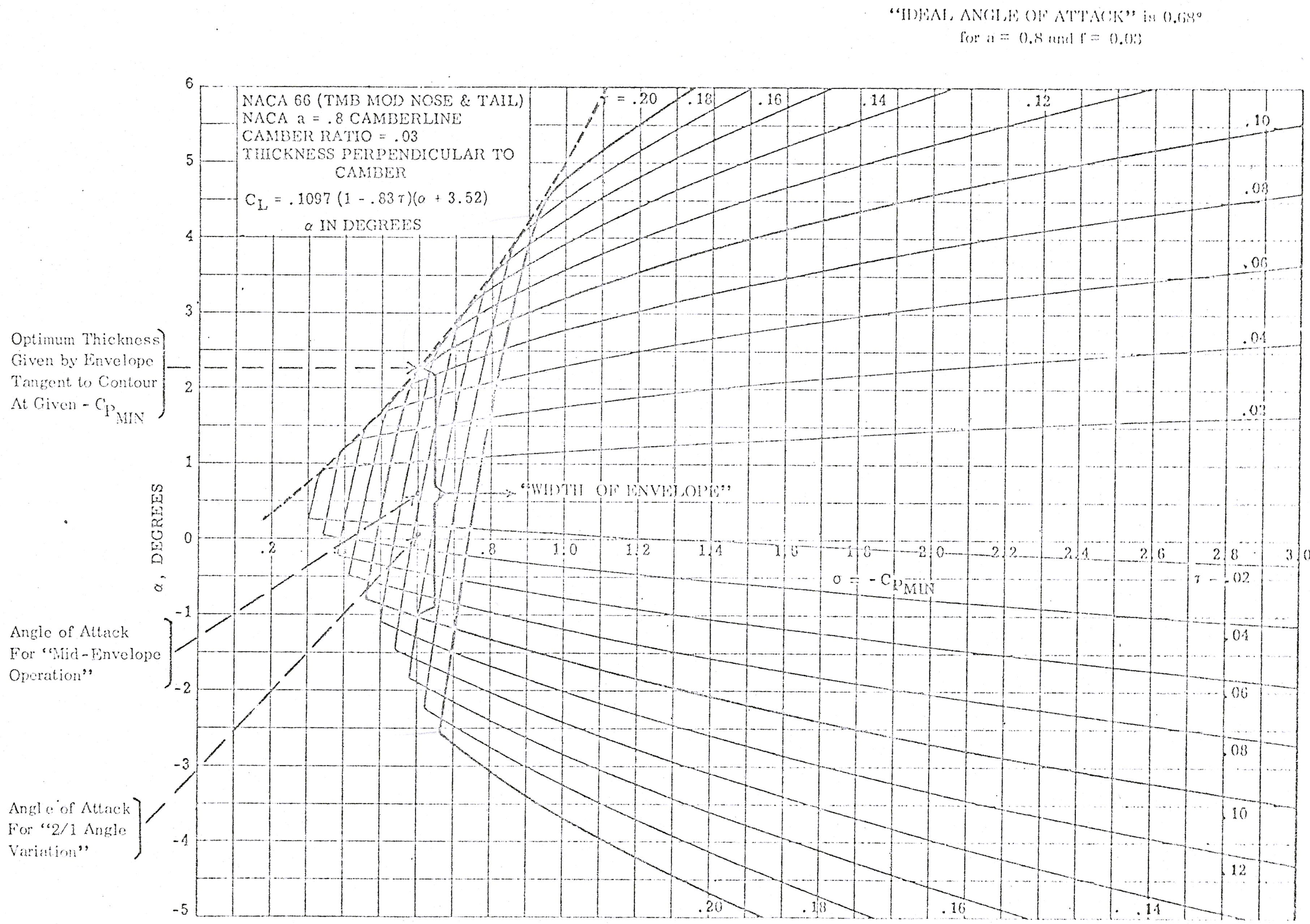
*Abbott and von Doenhoff, Reference 12, p. 70.



Minimum Pressure Envelopes for NACA 66 Sections (TMB Modified Nose and Tail) with the NACA a = 0.8 Camberline, Having a Maximum Camber Ratio of 0.03

FIGURE III.2-17

positive pressure shown in Figure III.2-16, which is right at the leading edge for an angle of attack of 0.68° and is on the lower face for more positive angles of attack, moves with negative angle of attack on to the upper face. A suction peak appears on the lower face and, for an angle of attack less than about -0.8°, the minimum pressure occurs here. This is the reason for the knuckle in the lower part of the envelope in Figure III.2-17, beyond which the pressure reduction increases rapidly for decreasing angle of attack. Brockett's envelopes are seen to define the critical cavitation index indicating the conditions of speed and submergence at which cavitation will begin for any angle of attack, and to border an open area to the right indicating cavitation free operation for a section of particular design.

III.2.1.5.b.ii Design for Maximum Speed

If a bounding contour be drawn tangent to the minimum pressure envelopes in Figure III.2-17, then the points of tangency define a relation between the thickness and the minimum pressure coefficient. Brockett has termed the intercept at the corresponding pressure coefficient the "width of envelope" for this thickness. For a given cavitation index, hence allowable minimum pressure coefficient, the corresponding thickness provides a maximum (or near maximum) width of envelope.

If the thickness is chosen in this way, for the cavitation index corresponding to the maximum foilborne speed, and the required lift coefficient corresponds to the angle of attack at the mid-point of the width of envelope, then there is a small margin of pressure in this design condition. Furthermore, an increase -- or decrease -- of angle of attack, due to wave action or control activity, equal to half the width of envelope may be tolerated before cavitation occurs. This, then, is referred to as mid-envelope operation. Design for a lower lift coefficient and angle of attack will provide a greater allowance for increased angle of attack and a lesser allowance for decrased angle of attack. An example of 2/1 angle variation is indicated on Figure III.2-17.

For the sections shown on this figure, all with a maximum camber ratio of 0.03, mid-envelope operation corresponds to a lift coefficient of about 0.4, depending on the cavitation index and resulting thickness. In order to provide a basis for design to any desired lift coefficient, diagrams have been prepared covering the whole range of Brockett's calculations. Figure III.2-18 shows the thickness corresponding to mid-envelope operation at a given cavitation index and lift coefficient. The corresponding width of envelope is also shown. The same information is given in Figure III.2-19 for operation with 2/1 permissible angle of attack variation.

From these diagrams it is clear that, at any design cavitation index, there is a maximum thickness even if no lift is to be developed. With decreasing thickness, a higher lift coefficient can be tolerated. At the same time the width of envelope decreases. If a smaller thickness is used at a given lift coefficient, then a greater margin is provided at the design speed to cover the effects of manufacturing errors, for example. The width of envelope is reduced, however.

The width of envelope, which is shown as a range of angle of attack in degrees, can also be interpreted as a range of lift coefficient by noting that a 1° increase of angle of attack will produce an increase of lift coefficient of about 0.1. The acceptable range of lift coefficient* may be derived from specifications on the allowable vertical accleration in a seaway. Note that this leads to a requirement for a percentage variation of lift coefficient, which is more easily achieved if the design lift coefficient is lower.

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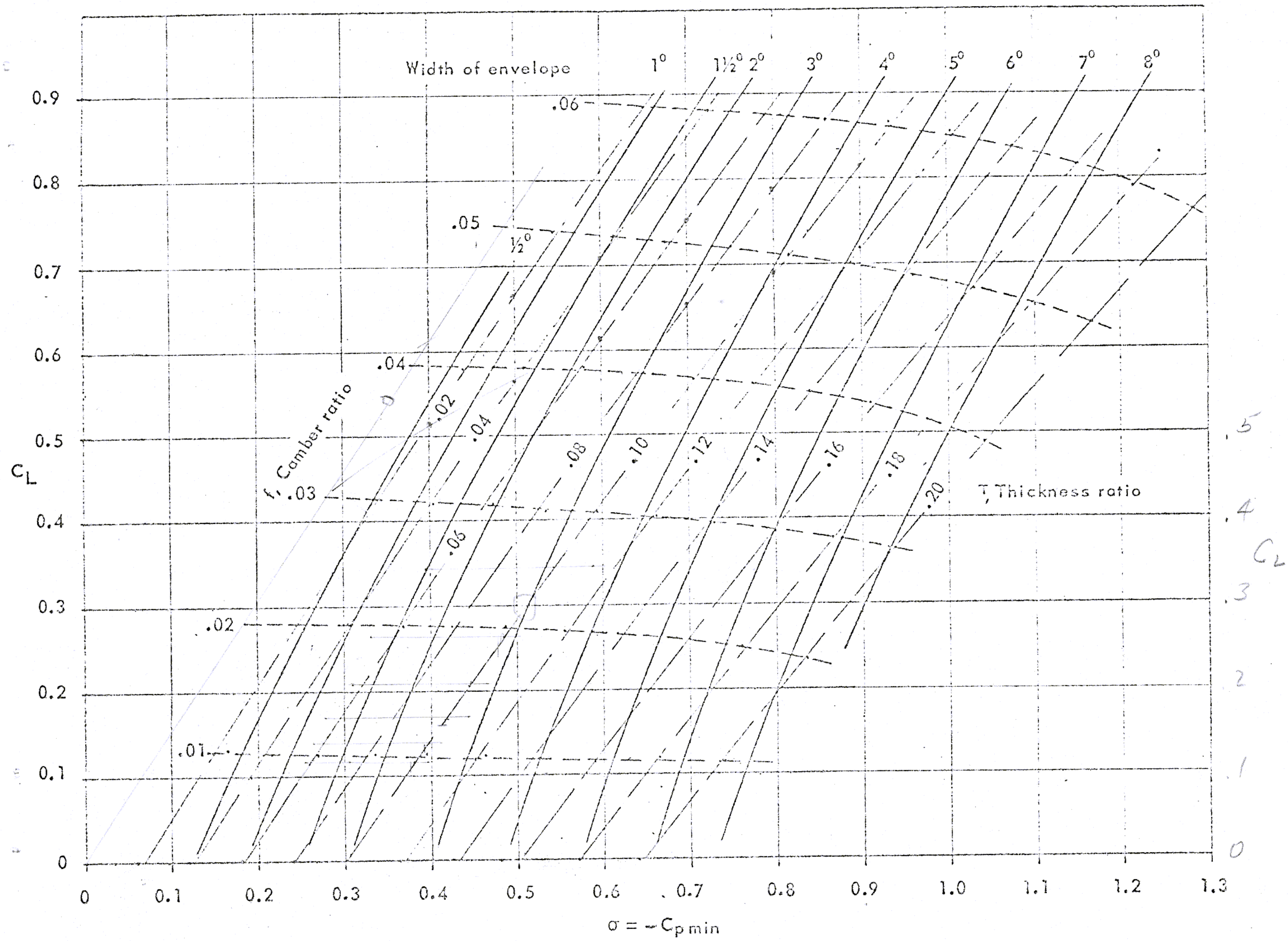
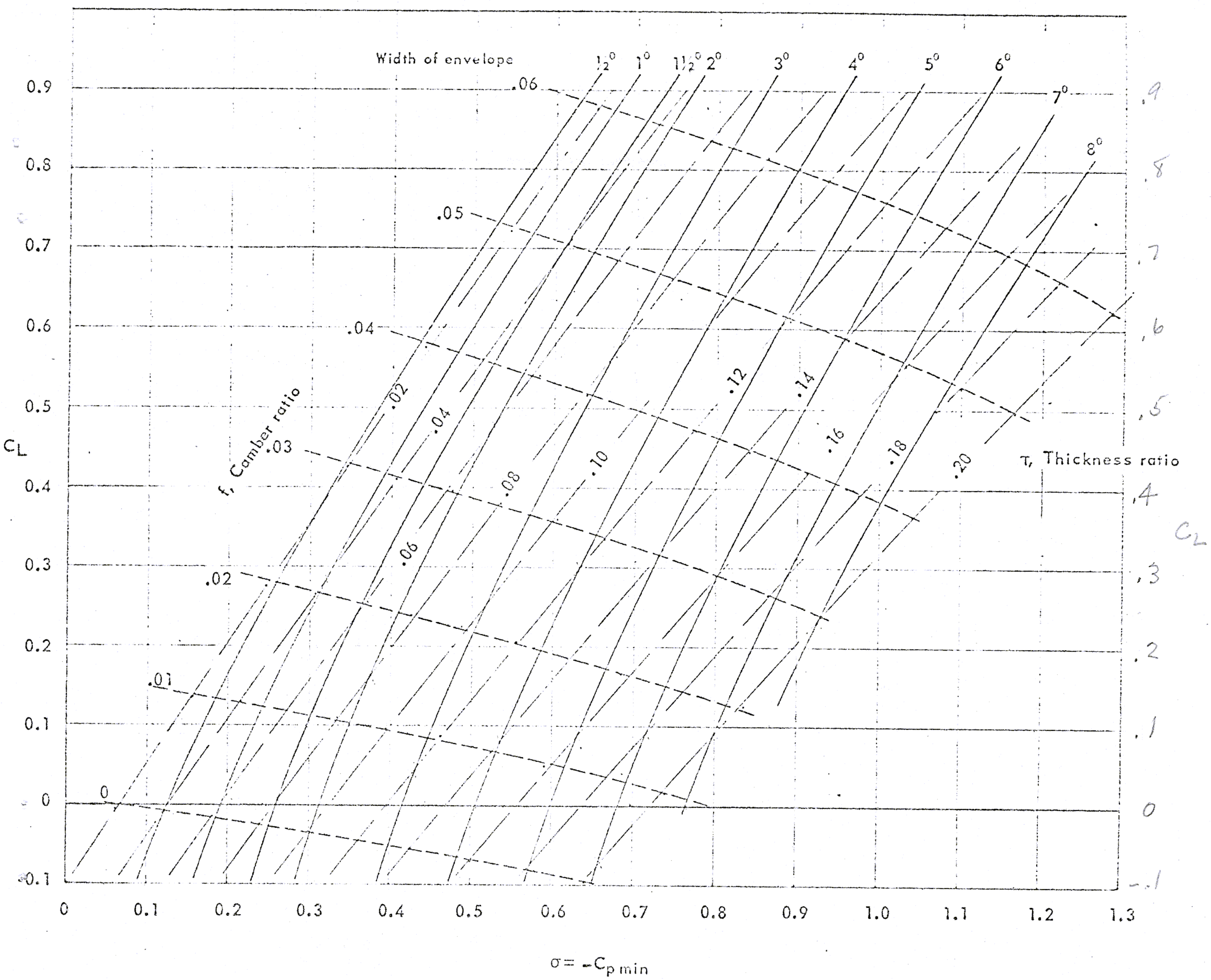
*An attempt to estimate the range of angle of attack from the wave orbital velocities would require corrections for the craft response and for the incidence changes produced by the control system and also take account of the induced angle of attack corresponding to the foil aspect ratio.



DESIGN OF CAVITATION-FREE FOILS; MID-ENVELOPE OPERATION
NACA 66 SECTIONS (DTMB MOD.) WITH $a = 0.8$ MEANLINES
(DERIVED FROM DTMB REPORT 1780, BY TERRY BROCKETT)

FIGURE III.2-18



DESIGN OF CAVITATION-FREE FOILS; 2/1 ANGLE VARIATION
NACA 66 SECTIONS (DTMB MOD.) WITH a = 0.8 MEANLINES
(DERIVED FROM DTMB REPORT 1780, BY TERRY BROCKETT)

FIGURE III.2-19

For reasons set forth in Section III.2.1.3.b it is preferable to consider a section normal to the sweep line when designing a foil to avoid cavitation. The cavitation index and the required lift coefficient should then be referred to the velocity component normal to the sweep line, which is $V_m = V_s \cos \varphi$ where φ is the sweep angle. The thickness and camber ratios then determined will refer to the length of the oblique section, which is

$$c' = \frac{c \cos \varphi}{1 - \frac{1}{2}\rho - \frac{3}{16}\rho^2}$$

where:

- c is the chord, measured parallel to the centerline, which intersects the oblique section at the 1/4 chord locus.
- φ is the sweep angle
- $\rho = \frac{2(1 - \lambda)}{AR(1 + \lambda)} \sin 2\varphi$
- λ is the taper ratio
- AR is the aspect ratio.

Due account must be taken of the variation of the lift coefficient along the span as determined by the planform, Section III.2.1.3, and by the downwash from any forward foil. The design must be based on the maximum local section lift coefficient, rather than the mean value.

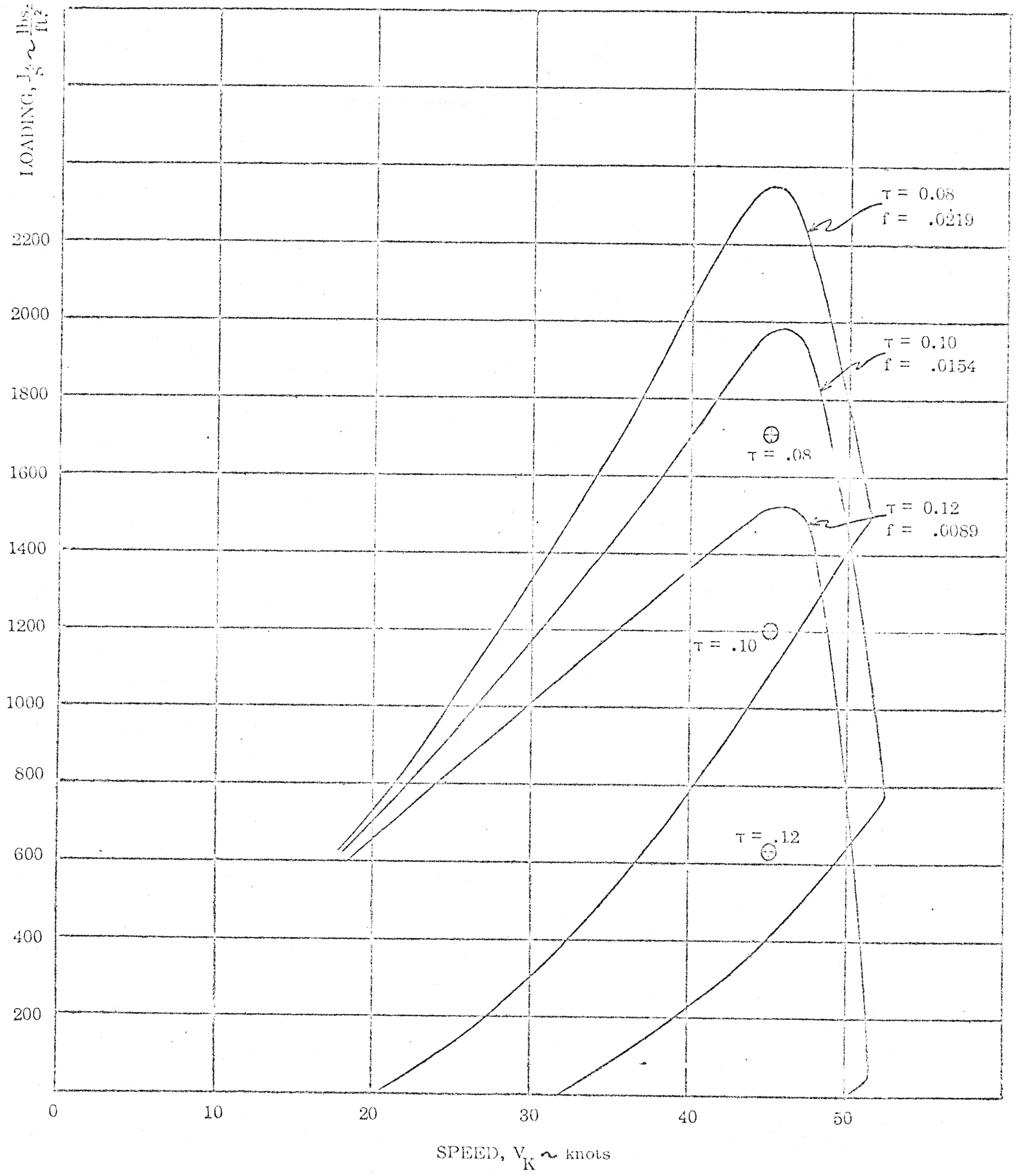
III.2.1.5.b.iii <u>Off-Design Speed Operation</u>

When the speed of the craft is reduced, the lift coefficient must increase since the required lift remains constant. Thus either the angle of attack must be increased or, if flaps are fitted for control, the flap deflection must be increased. In the absence of flaps the pressure distribution is adversely affected, as is shown in Figure III.2-16, for example, and cavitation may occur in the region of the suction peak on a foil which is free of cavitation at the design speed.

The plot in Figure III.2-20 illustrates the effects of speed variation. Curves are shown for three different sections, all designed on the basis of Brockett's data to have the maximum width of envelope for a speed of 45 knots and a submergence of $7\frac{1}{2}$ feet ($\sigma = 0.446$), and indicate the range of allowable loading at any speed. The mid-envelope point is also shown for each at 45 knots. For any particular foil the loading, L/S, will remain constant when the speed is varied.

It is evident that a larger range of cavitation-free speeds is available when a lower design loading is used. Furthermore a design to somewhat less than the mid-envelope point reduces the minimum allowable speed. This is another reason, in addition to the low speed drag considerations noted in Section III.2.1.2, why in most cases the foil area cannot be selected to provide minimum drag at maximum speed.

The figure referred to above is based on the NACA 66 series of foil sections, as modified by the David Taylor Model Basin. The original purpose of the development of these sections was to achieve a favorable chordwise velocity and pressure distribution, over as much of the chord as possible, at the design lift coefficient. For operation at off-design speeds, and from the point of view of cavitation avoidance, a section with a larger nose radius might be preferable. The modified NACA four-digit sections, typified by the NACA 0010-65 basic thickness form with



CAVITATION "BUCKETS" FOR FOILS DESIGNED FOR 45 KNOT OPERATION AT 7½' SUBMERGENCE

NACA 66 SECTIONS (DTMB MODEL), a= 0.8 MEANLINES

⊕ -- MID-ENVELOPE POINTS

DATA FROM DTMB REPORT 1780

FIGURE III.2-20

suitable camber, offer possibilities in this direction*. Unfortunately there are not available at the present time minimum pressure curves for such a section, comparable to Brockett's curves for the NACA 66 series sections.

The foregoing discussion of operation at off-design speeds is predicated on the use of angle of attack variation to control lift. If flaps are used for lift control it will, in general, be possible to operate over a wider range of speeds without incurring cavitation because a better distribution of pressure can be maintained. This is illustrated in Figure III.2-21 which compares the pressure distribution over a symmetrical foil at an angle of attack with that over the same foil at zero angle of attack but with a 27% chord flap deflected just enough to give the same lift coefficient. For a cambered foil with a flap, there will be an optimum combination of angle of attack and flap deflection for any required lift coefficient. Thus, to achieve the maximum benefit from flaps, it is necessary to be able to adjust the effective incidence of a flapped foil, either by varying the pitch trim of the ship or by means of a separate incidence control on the foil. This requirement will have an important influence on the design of the control system.

Unfortunately, there are not available contours of the minimum pressure coefficient, similar to Figure III.2-17, for foils with flaps. A method for predicting the chordwise pressure distribution on flapped airfoil sections is given in Reference 13 and was used in the preparation of Figure III.2-21.

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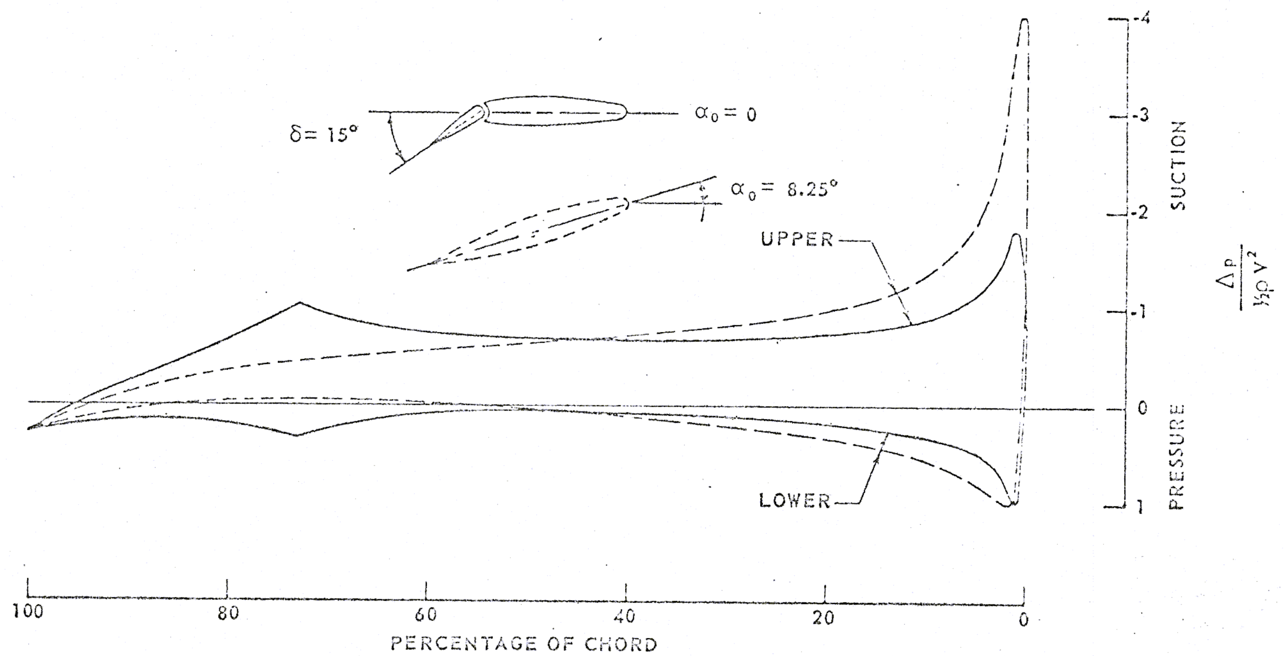
*Compare, for example, the values of $\Delta v_a/V$ for this section with those for the NACA 66-010 section (pp. 319 and 375 of Reference 12).



CALCULATED PRESSURE DISTRIBUTIONS ON NACA 16-012 AIRFOIL WITH AND WITHOUT 27% CHORD FLAP, $C_l = 0.91$

FIGURE III.2-21

III.2.1.5.b.iv Section Design for Swept Foils

The effect of sweepback on the load distribution over the foil surface was noted in Section III.2.1.3.b. It was suggested that, if the aspect ratio were sufficiently large, the foil could be considered in three parts: (1) a center or kink region extending approximately 1/2 chord length each side of the center section; (2) a tip region within about 1/2 chord length of the tip; and, (3) a sheared region between the center and tip regions. If the aspect ratio and the taper ratio are low enough, the center and tip regions may overlap with no appreciable sheared region.

. Over the sheared region the load may be assumed to be distributed chordwise like that on a foil of infinite aspect ratio. Thus the two dimensional data cited in the previous section is applicable, bearing in mind that the pertinent velocity is the component of the ship speed normal to the sweep line. The controlling geometry is that of sections cut by planes normal to the sweep line, with the angle of attack modified by the factor sec γ as described in Section III.2.1.3.b. The maximum local lift coefficient in this region will govern the choice of section characteristics. Furthermore, in examining the off-design speed operation, the additional lift coefficient must be calculated on the basis of the planform without redistribution due to twist. That is to say the basic spanwise loading, in nondimensional form, is independent of speed while, at a lower speed, the added loading* will have a more significant effect on the total span loading.

.. In the vicinity of the kink at the center of a swept foil the effect of the sweepback is to reduce the spanwise loading, which of course increases the loading over the rest of the foil. Coincidentally the load is shifted rearward along the chord near midspan. It has been pointed out in Section III.2.1.4.a that the spanwise loading can be adjusted by the use of twist[2,3].

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*In the sense of Reference 2.

The method of Reference 2, however, gives no clue to the chordwise load distribution. In fact the validity of the procedure of de Young and Harper[2], and of the theory of Weissinger[4] on which it is based, is seriously questioned by Thwaites[1] and by Kuchemann[5] for their dependence on a two dimensional, flat plate chordwise load distribution. The method continues to be used because of the ease of computation, and gives quite satisfactory results for the spanwise loading[3].

The importance of the chordwise load distribution in the center region depends on the spanwise loading or, more particularly, on the spanwise variation of the lift coefficient. Thus, if the foil is not twisted and similar sections are used over the whole span, then the minimum pressure will not occur at the center but at the tip*. The center region of the foil will be shirking its job, but there need be no concern over the chordwise loading. If, on the other hand, the foil is twisted to achieve a more nearly uniform spanwise distribution of the lift coefficient, then the chordwise load distribution in the center region must be carefully considered.

This problem has been extensively studied by Cohen,[6] Kuchemann[5], Brebner[7] and Weber[8] and reviewed in considerable detail by Thwaites[1]. Cohen has given a particularly illuminating illustration of the effects, but her calculation method is tedius and probably impossible to computerize since it involves several graphical steps. Kuchemann's method depends on the use of several interpolation functions which were largely intuitively derived. His results have been experimentally verified for symmetrical sections and the method extended to cambered sections by Brebner.[7] The results of primary interest here are shown in Figures 20 and 21 of Reference 5. Briefly, it is shown that, if symmetrical

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*See, for example, Reference 1, Figure VIII.18 for flat foils and Figure VIII.31 for foils with parabolic camber.

sections are used, then it is necessary to use both twist and (negative) camber to preserve the two-dimensional chordwise load distribution at the center of the foil. When the uniform-load camber line (NACA a = 1.0 mean line) is used, then the use of twist alone is nearly adequate to maintain the two-dimensional chordwise loading. It seems safe to assume that, when using the NACA a = 0.8 mean line, the chordwise load distribution at the foil center will be quite satisfactory if twist is used to acquire the desired spanwise load distribution.

... Near the tip of a swept foil the span loading is increased compared with an unswept foil. The chordwise loading is moved forward as is the velocity distribution pattern due to thickness. These features have been likened by Thwaites[1] (p. 325) to the situation at the center of a foil with forward sweep. This does not mean that the span loading is the same at tip as in the center of a foil of opposite sweep, but the change from that on an unswept foil is comparable, as has been noted in Figures III.2-6(a) and (c). The influence of sweep on thickness effects is extensively discussed by Thwaites[1] (p. 284) who proposes that a factor of 0.7 be applied to the velocity increase calculated for the center of a swept forward foil.

So far as the chordwise load distribtuion is concerned, the analogy between the tip of a swept back foil and the center of a foil with forward sweep[1, 5] is not clear to this writer, since the configuration of the vortices by which the lifting foil is represented are so different in the two cases. If this is a true representation, then the loading is distributed nearer to the leading edge and the minimum pressure point will also be nearer the leading edge.

It is important then to avoid excessive spanwise loading near the tips, which is the reason for twisting the swept foil in the sense to unload the tips. With twist, and with a camber line such as the NACA a = 0.8 mean line, it appears reasonable to expect that the pressure distribution will approximate reasonably well to the two-dimensional distribution for the section chosen, at the design speed.

In order to delay cavitation at reduced speed, where increased angle of attack must be used and the added load is distributed disproportionately near the tips, the camber may be somewhat increased -- and also the twist -- so that the tip sections are operated, at design speed, at a lower point in the minimum pressure envelope. Even with constant camber the reduction of lift coefficient toward the tip has this effect.

III.2.1.6 TIP SHAPE

The shape of the foil tip involves details of the planform and also the character of spanwise sections at the tip. The geometry of streamwise sections near the tip is also necessarily involved.

Variations of tip shape can affect the foil drag, the strength of the tip vortices and the inception of cavitation on the foil.

- Drag

Classical wing theory[5] indicates that an elliptical spanwise load distribution will lead to a minimum of the induced drag and that this will result from an elliptical planform and spanwise uniform sections on a straight wing. Approximations are made in the development of this theory, however, which are known to be increasingly inexact near the tips. In fact, the elliptical loading is not maintained all the way to the tip of a wing with elliptical planform. Instead the loading falls off more rapidly and the wing behaves as if the span were smaller than the actual span. The use of squared off tips -- in plan -- with sharp corners has been shown[21] to increase the effective span and, hence, to reduce the induced drag. In particular the trailing edge should be carried straight out to the tip. Thinning of the tip to a sharp edge appear to contribute to this purpose, though a flat end is almost equally effective.

- Wake

As has been noted in Section 2.1.4.c the pattern of the wake and downwash field behind a foil is characterized by a distribution of trailing vorticity with a concentration of strength into tip vortices at the edges. The degree of concentration, at any downstream position, is influenced by the spanwise load distribution. Consequently the use of square cornered, sharp edged tips suggested above for drag reduction, will increase the strength of the tip vortices and make it more important to avoid their inpingement on an after strut or foil.

. Cavitation

On a tapered, swept foil the added lift coefficient necessary at reduced speed tends to be concentrated toward the tips as is shown in Figure III.2-6(b). Under these conditions a tip detail which further concentrates the span load in the tip region may not be desirable. The situation at low speeds and at takeoff should therefore be examined with some care.

The possibility of the development of tip vortex cavitation under high lift conditions is illustrated in Reference 22, along with cavitation over a considerable part of the foil near the tips. Since such operation would be only transient, this is probably not a serious consideration unless control difficulty is experienced. Model tests appear to be the only possible method of examination and should include tests at a yaw angle to induce asymmetrical loading.

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