



MANDLES PRIZE FOR HYDROFOIL EXCELLENCE 2022

THE INTERNATIONAL HYDROFOIL SOCIETY



Velocity Prediction Program for a Hydrofoiling Lake Racer

Michele Francesco Melis (Author), Hamburg University of Technology, Hamburg, Germany
[michele.melis@tuhh.de]

Prof. Dr. -Ing. Moustafa Abdel-Maksoud (Faculty Adviser), Institute for Fluid Dynamics and Ship Theory at the Hamburg University of Technology, Hamburg, Germany
[m.abdel-maksoud@tuhh.de]

ABSTRACT

The aim of this paper is to develop an accurate and robust six degrees of freedom stationary VPP model applied to a high-performance sailing yacht. The model is set up to assist NC Raceboats with the VPP based hydrofoil design, considering the sailing performance in three modes: archimedean, transition and hydrofoil. The yacht is a lightweight monohull designed for light wind conditions with a variable number of crew members. The design includes a self-stabilizing hydrofoil configuration and an elevator rudder. The software tool, which is used for the velocity prediction program, is FS-Equilibrium, developed by DNV. The software offers a modular workbench in which each force can be modelled with semi-empirical force modules, which are based on validated methods and theories. The performance prediction are interpreted and discussed: as foreseen, the performance of the high-performance lake racer in hydrofoiling condition is significantly greater compared to its assessment in archimedean sailing mode. In medium breeze conditions, the yacht is able to lift up on its hydrofoils and attain flight mode. The minimum hydrofoiling speed investigation demonstrates that the VPP is able to consistently iterate through the transition mode. This paper shows that it is possible to develop a VPP model for a hydrofoiling sailing yacht on the basis of relatively simple assumptions and theories.

NOTATION

AoA	Angle of Attack ($^{\circ}$)
CFD	Computational Fluid Dynamics
DSYHS	Delft Systematic Yacht Hull Series
TWA	True Wind Angle ($^{\circ}$)
TWS	True Wind Speed (m/s)
VMG	Velocity Made Good (m/s)
VPP	Velocity Prediction Program
Vs	Boat Speed (m/s)
$1 + k$	Form Factor
<i>Elevator</i>	Angle of Attack Elevator Hydrofoil ($^{\circ}$)
<i>ElevatorImmersion</i>	Sinkage Elevator Hydrofoil (m)
<i>Flat</i>	Sail Trim Variable
<i>Heel</i>	State Value Heel Angle ($^{\circ}$)
<i>Long – crew</i>	Crew Longitudinal Movement
<i>Lat – crew</i>	Crew Lateral Movement
<i>LiftFraction</i>	Lift Portion Ratio
<i>MxCtrl</i>	Heeling Moment Control Variable
<i>Rake</i>	Angle of Attack Surface-piercing Hydrofoil ($^{\circ}$)

INTRODUCTION

Since the very first sailing competitions, the aim of every competitive sailor has always been to sail faster. For this reason, designers sought to improve this performance on the engineering side. The design of a sailboat is a complex process and when it comes to racing sailboats on hydrofoils the quest for high performance increases. It is easily deducible that the more complex the sailboat is, the more highly sophisticated software programs have to be developed in order to model its behavior and performance accurately. For example, an excellent tool is the velocity prediction program, which offers advantages for design optimization: benefits in righting moment with drawbacks in drag, gain or loss of side force etc. An important challenge for designers is to maximise speed by minimizing drag and displacement: design a way to partially or completely support the weight of a sailboat on so-called hydrofoils. Through recent breakthroughs of this new technology in the America's Cup, many of the newest high-performance sailing yachts have adopted a hydrofoil configuration. Among numerous new designs, the lake racer is a high-performance fast and light wind flier designed for the world's most famous lake regatta, the Bol D'Or Mirabaud held in Geneva, Switzerland.

THE LAKE RACER

The sailboat, displayed in figure 1, is a highly technological lake racer designed by Hugh Welbourn for competing on flat lake waters. The yacht has been designed as an all-round non-compromised sailing racer with a low safety factor, making it very light and able to quickly rise on its hydrofoils. With a total length of 10.5 meters and a beam of 4 meters, the total displacement is approximately 1300 kg including 4-5 crew members, depending on the required righting moment. The upwind sailing area reaches nearly 60-80 square metres and downwind roughly 120 square metres. The sailing regime varies from 0 up to 20 knots TWS with three different sailing modes. In light airs, which are quite frequent on lakes, the yacht sails in the archimedean mode where the main hydrofoil is completely retracted out of the water and does not encroach the sheeting of the sails. When sailing in medium breezes, the appendage is semi-immersed creating righting moment for the skimming mode. When winds are stronger than 10 knots, instead, the appendage is immersed deeper in the water and the lake racer is supposed to fully hydrofoil. The flying configuration, illustrated in figure 1, demands two non-symmetrical self-levelling surface piercing hydrofoils. Their task is to create lift on the leeward side and, while manoeuvring, also on the windward side. The crew is sitting on the side deck and hiking to windward. The hydrofoil, the keel bulb and the crew ensure the required stability. The pitch angle of the yacht is adjusted by the rudder elevator and the crew's position. In order to balance the side force generated by the sails, the hull is equipped with a keel fin.



Figure 1: Appendage Configuration (a) and Hydrofoiling Yacht (b) [source: www.sail-world.com/news/240734 (15.10.2021)]

In practice, reaching and maintaining an equilibrium while hydrofoiling depends on various boat parameters that need to be regulated simultaneously. This feature must be mirrored in the VPP model as well. Given a self-stabilizing hydrofoil configuration, the main task of this paper is to predict and optimize boat speed in hydrofoil sailing mode, while accounting for the conditions in which the minimum hydrofoiling speed occurs.

VELOCITY PREDICTION PROGRAM

A VPP is currently one of the most important tools available to sailors and designers in professional sailing. The program estimates the performance of a sailing yacht given certain boat model and environmental data through a solution algorithm. The latter interrogates the model by new input values until certain conditions are fulfilled, for example when a force equilibrium is reached. Figure 2 provides an overview of the graphical user interface for the software tool FS-Equilibrium. Since the program was first developed (Hochkirch, 2000), it has continually been advanced. FS-Equilibrium works as an open modular workbench and is capable of analysing stationary and instationary sailing states. A detailed description of the software functionality including algorithms, formulae, flow charts of employed models can be found in the user manual (DNV, 2021) and in VPP applications for a hydrofoiling C-Class catamaran (Paulin, et al., 2015) and for a AC50 (Hansen, et al., 2019).

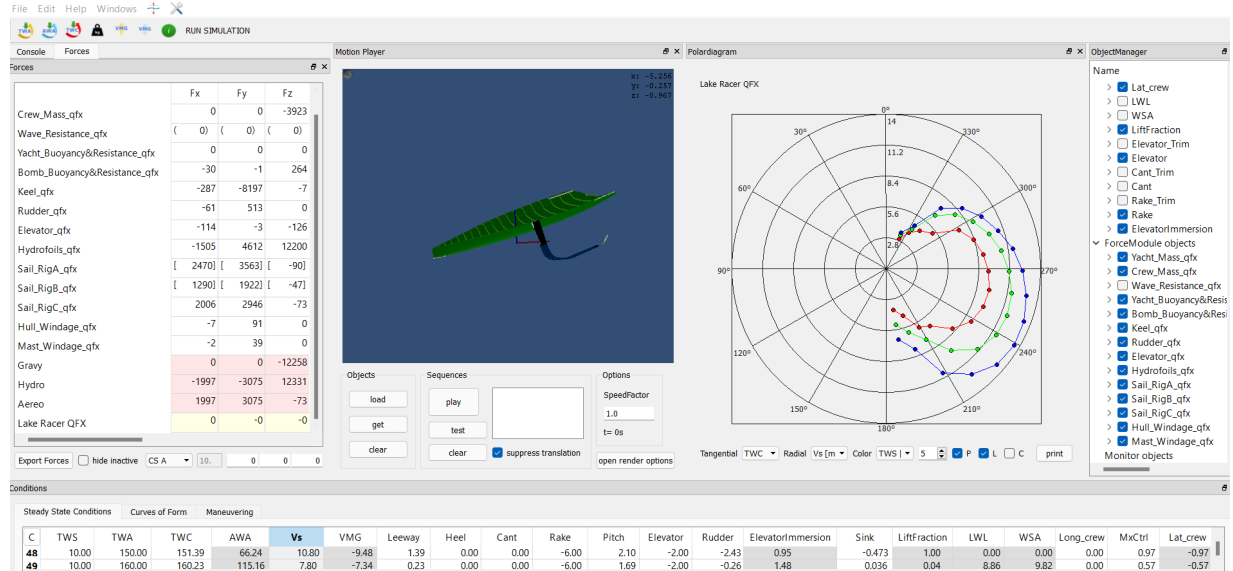


Figure 2: Graphical User Interface of the Velocity Prediction Program in FS-Equilibrium

The program allows to define the forces that act on the boat model with so-called force modules. Numerous types of force modules have been implemented based on theoretical considerations, semi-empirical or numerical methods, or input from experimental or numerical simulations. For this project, the utilized force modules can be divided into three categories: gravity modules, hydrodynamic or hydrostatic modules, and aerodynamic modules. Table 1 provides an overview of the employed force modules, along with a description of which force they model. A detailed description of each force module and its method is provided further below in this report.

Table 1: Forces Modules implemented in the Velocity Prediction Program for the Lake Racer

Force Type	Module Name	Module Type	Remark
Gravity:	Yacht Mass	Mass	Weight force of the fully rigged hull
	Crew Mass	MoveableMass2D	Weight force of the five crew members
Hydrodynamic:	Wave Resistance (Inactive)	DSYHS 2008	Wave system resistance force of the upright hull
	Yacht Buoyancy & Resistance	Buoyant Force	Buoyant and viscous forces of the submerged hull
	Bomb Buoyancy & Resistance	Canting Object	Buoyant and viscous forces of the keel bulb
	Keel	Low Aspect Foil	Lift and drag forces of the centerboard keel fin
	Rudder	Low Aspect Foil	Lift and drag forces of the transom hung rudder
	Elevator	Low Aspect Foil	Lift and drag forces of the elevator hydrofoil
Aerodynamic:	Hydrofoil	Lifting Line Appendage	Lift and drag forces of the rounded hydrofoil
	Sail Rig A	IMS Rig 2003	Sail forces for main sail, jib and spinnaker
	Sail Rig B	IMS Rig 2003	Sail forces for main sail and working code0
	Sail Rig C	IMS Rig 2003	Sail forces for main sail and light code0
	Hull Windage	FWindage	Parasitic drag force of the unsubmerged hull
Mast Windage	FMast Windage	Parasitic drag force of the tapered mast	

In the setup of the software, various parameters can be defined. *State values* describe the position of the sailing yacht, *trim values* can optimize its performance. In FS-Equilibrium two algorithms are employed in different loops: the first one searches the equilibrium condition by adjusting the state variables, while the second is employed in an outer optimization loop searching for maximum speed changing the defined set of trim values. Different algorithms are available and can be selected based on the application. Within the stationary mode, the program finds the equilibrium for a specified wind condition using the gradient-based Newton-Raphson algorithm. While solving the equation of motions in the stationary mode, the sum of the external forces and moments adds up to zero within the six degrees of freedom. The outer optimization uses the non-gradient based Hooke and Jeeves method (Hooke and Jeeves, 1961).

Gravity Force Models

These modules express the total forces of gravity acting on the lake racer. They include the respective mass and mass center of the fully rigged lake racer and crew members. The crew movement has to be considered in the model, since especially for the light weight lake racer, it has a strong impact on the yacht's performance. The longitudinal movement, modelled in the VPP with the trim parameter *Long-crew*, affects the pitch of the boat, especially while hydrofoiling. The crew members hiking on the trapeze, modelled with the trim parameter *Lat-crew*, provide righting moment and hence less heel angle for the yacht's equilibrium. These parameters can be optimized or linked to other variables controlling the performance.

Hydrodynamic Force Models

According to its design requirements, the yacht must perform outstandingly in each sailing mode: displacing, planing and skimming through the transition into hydrofoil mode. Predicting the exact hydrodynamic bare hull resistance in all states is quite challenging. The light weight and flat underwater part of the hull indicate a planing yacht. Therefore, when a certain speed has been reached, a considerable portion of the weight is supported by the hydrodynamical lift acting on the flat bottom. For the bare hull of the lake racer no resistance measurements are provided. An empirical approach is therefore required. In order to model this resistance, the following two approaches are considered and compared during the setup. The first is to calculate the viscous resistance in the buoyancy force module following the correlation line (ITTC, 1957) and the residual resistance using a Delft Systematic Yacht Hull Series force module (Keuning and Katgert, 2008). The second approach relies exclusively on computing the viscous resistance scaling it with an empirical factor to account for the wave system resistance.

The buoyancy force module describes the buoyant and viscous forces and generated moments acting on the submerged part of the canoe body hull. The total displacement of the lake racer can be expressed with one or more buoyancy modules. For instance, the buoyancy and the viscous forces of the keel bulb are not included here but are instead modelled by an other force module. The imported canoe body hull is divided by the force module in sections which are then used for the calculations. By means of a strip theory approach the hydrostatic forces and wetted surface area of each section are calculated. From the sum of the forces and each panel position, the heel and pitch moments are obtained. The center of buoyancy is then determined from the resulting moments. The implemented correlation line computes the viscous resistance based on the actual wetted surface area, the form factor $(1 + k)$ and the Reynolds number, which uses a variable waterline length as reference. The calculated viscous force acts in the center of buoyancy and hence results in further pitch, heel and yaw moments acting on the hull. Considering only the submerged sections, the buoyant and viscous forces are computed in every iteration: if the hull rises a certain amount out of the water, the buoyant forces decrease and so does the viscous force. Being able to display this dynamic behavior for the yacht is profitable to predict its performance in all sailing modes, from the archimedean through to transitioning into hydrofoiling. In order to monitor the influence of the hydrodynamic lift, a derived value is defined as *Lift Fraction*, which is the total mass portion that is carried by the hydrofoil lift. As displayed by figure 3, if close to 0, the lake racer is floating in the archimedean sailing mode. If close to 1, instead, the lake racer is skimming in the transition mode. If *Lift Fraction* equal to 1, it is completely lifted out of the water and therefore hydrofoiling.

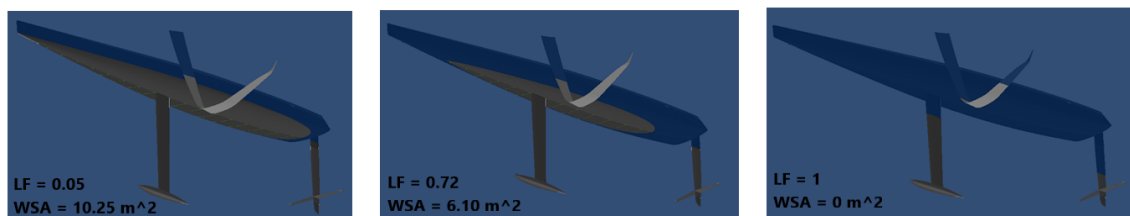


Figure 3: Change of *Lift Fraction* and Wetted Surface Area Bare Hull

The Delft series wave resistance force module uses empirical regression formulas to calculate the upright and heeled wave system resistance of the bare hull. These are only formulated for fast displacement sailing yachts and not for planing hulls. The force module does not account for the dynamic change of displacement and wetted surface area of the yacht while reaching higher boat speeds. Considering the case of the hydrofoiling lake racer, the wetted surface area changes significantly when transitioning into planing and hydrofoiling. In this condition, the effective displacement becomes minor and consequently, the residuary resistance is almost negligible. These effects cannot be accounted for by the Delft Series wave resistance. According to its design, the lake racer is expected to transition rapidly into planing mode, reaching high speeds already in light winds and even higher speeds when adopting the hydrofoiling configuration.

Figure 4 provides a comparison between the two considered approaches in modelling the bare hull resistance. The forces and boat speeds are exported from a VPP run of the hydrofoiling lake racer for TWS of 5 m/s and TWA 90 degrees. When the boat speed is between 1 and 3.5 m/s, the scaled ITTC approach slightly overpredicts the total resistance. The respective speed divergence is less than one knot, which is acceptable for the scope of this project. Up to a boat speed of 4.8 m/s, the lake racer is expected to sail in its archimedean configuration. For this regime, the blue and red curves match, which means that the divergence prior to the transition mode is marginal. In this condition, with boat speeds of approximately 5 to 6.5 m/s the *Lift Fraction* is greater than 0 but less than 1. Even if a resistance hump is computed, the Delft series force module, cannot model the hull resistance adequately in the transition and hydrofoiling modes as the reduction of displaced volume and wetted surface cannot be accounted for. For this reason, the bare hull resistance is modelled exclusively with the viscous resistance using the actual wetted surface area with the higher form factor. This bare hull resistance contains the frictional resistance, viscous pressure resistance and an increase for taking into account the wave system resistance.

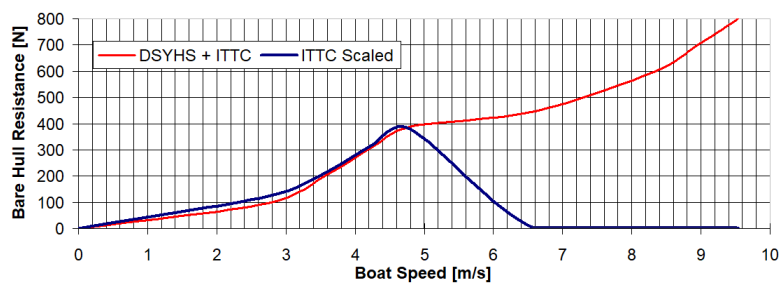


Figure 4: Comparison of Hydrodynamic Modules for Bare Hull Resistance

The "low aspect foil" force module approximates the low aspect ratio effects and calculates the total lift and drag of a three-dimensional appendage arbitrarily positioned on the sailboat, in this case the keel, rudder and elevator of the lake racer. The forces calculation follows the systematic analysis of movable control surfaces (Whicker and Fehlner, 1958). The lift and drag coefficients are computed using cross-plots of the characteristic parameters based on experimental work and potential theory. The computation of viscous forces refers to the correlation line (ITTC, 1957). With this force module, it is possible to model vertical and horizontal control surfaces which can be fixed or moveable. The loading distribution of these surfaces is determined by a specific efficiency factor. The adjustment of the elevator surface is controlled by the trim value *Elevator* angle, which varies between 5 (for positive upwards lift) and -7 degrees. The parameter regulates the amount of rudder lift and influences the pitch angle of the lake racer. Depending on how the lake racer is sailed, this variable can be optimized or fixed to a value. The interaction of changing this parameter and steering the rudder is not considered by this force module.

The forces generated by the surface-piercing hydrofoils are modelled by implementing a lifting line appendage module, which refers to the Prandtl lifting line theory (Prandtl, 1923). In the force module, the interaction effects of numerous hydrofoils can be considered, as long as the wake vortices or the respective lifting lines do not intersect each other. In order to attain a robust velocity prediction for the lake racer in foil borne condition, NC Raceboats provided an initial draft design along with CFD data for the appendage. According to the designer, the appendage is capable of lifting the total weight of the hull and crew at a boat speed of 6 m/s. This predicted minimum hydrofoiling speed will be verified in the VPP model. Regarding the degrees of freedom of the appendage, the vertical position is fixed for this VPP model, which means that the cant angle and the hydrofoil leverage arm (or righting moment) are limited. The rake angle, defined in the VPP as the trim value *Rake*, can vary between 6 and -7 degrees. Depending on the real handling of the lake racer, this value can be fixed or optimized. For computing the lift and drag distribution over the entire span, the force module requires the three-dimensional geometry of the hydrofoil, which will be simplified as a two-dimensional surface between the leading and trailing edge. Out of the complex total three-dimensional object, only one surface is required by the module to compute linear parametrization

in chord and span direction. In case of surface piercing hydrofoils, the root and the tip are mainly out of the water in order to ensure stability, adjusting constantly their wetted surface area. For this reason it is necessary to pay particular attention while considering the influence of waterline surface effects in order to obtain a valid output of the acting forces. The force module implementation allows only one intersection with the free surface per surface. The appendage surface is therefore split into two surfaces around the most submerged point: root surface and tip surface. In order to compute the lifting line operations only for the submerged portion of the hydrofoil, the two surfaces are cut at the free waterline intersection as displayed in figure 5. For high Froude numbers, the presence of free surface causes an increase in resistance while the upwards lift decreases. Therefore, to account for the residuary resistance of the surface-piercing hydrofoil in the VPP model, an anti-symmetrical vortex image above the waterline has been considered in the module following the approach suggested by (Faltinsen, 2009). This is often referred to as the biplane approximation and implies that the fluid velocity due to the hydrofoil is vertical at the free surface in high speed conditions. For both surfaces, the module defines a mirrored waterplane and computes the forces in the waterline coordinate system instead of the body fixed coordinate system. Subsequently, it transforms these back to display their effect on the yacht. In order to apply the lifting line method, the force module utilizes two vectors which divide both surfaces in panels, as displayed in figure 5. The lifting line calculations are performed on each panel obtaining results for sectional lift and drag forces. For each surface, the module requires the input of lift coefficient and zero lift angle as functions of span for the lift and induced drag distribution. For the viscous resistance it requires the drag coefficient as function of lift coefficient, span and Reynolds number. This drag component is calculated for each two-dimensional submerged panel and summed over the wetted span. According to two-dimensional airfoil theory (Abbott, 1959), the lift produced by a wing is proportional to the size of its span, to the square of its velocity, to the flow density and to the AoA of the wing to on-coming flow. Following this theory, it can be assumed that the variation of the lift coefficient, for small AoA , is approximately linear. The gradient is taken constant over the whole span and for both surfaces it is equal to 2π . The required zero lift angle as function of non-dimensional span is the AoA of each wing section for which the lift coefficient is equal to zero. The latter varies because of the designed hydrofoil twist. When the lake racer sails in hull borne condition, because of low boat speeds, the viscous resistance of the surface piercing hydrofoil has less influence and the induced drag prevails. Once it reaches the foil borne condition, instead, the induced drag decreases with the squared boat speed and the viscous resistance increases.

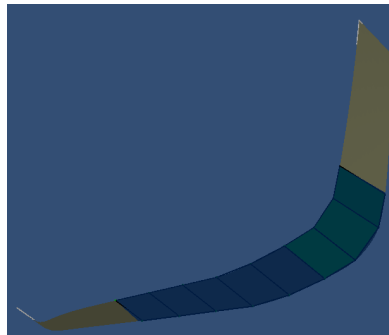


Figure 5: Lifting Line Cut of the Tip (Blue) and Root (Green) Surfaces at the Free Waterline Intersection

Aerodynamic Force Modules

In order to sail in various environmental conditions, the sail plan of the lake racer contains the following sails: main, genoa, working code0, light code0 and asymmetrical spinnaker. For these sails, none wind tunnel measurements or CFD analysis are provided. The aerodynamical forces are therefore modelled in the VPP using semi-empirical force modules. These forces are modelled by the IMSRig force module, which utilizes the International Measurement System VPP (ORC, 2001). This module uses two trim parameters, Reef and *Flat*, to optimize the sail trimming. The Reef parameter is not adjusted in the model, meaning that the sails remain fully hoisted on the mast. The *Flat* parameter reduces the lift coefficient, and consequently the induced drag of the sail while the parasitic drag remains constant. The module derives the lift and drag coefficients of each sail through default response functions based on IMS rules. These coefficients are then standardized and implemented in the force module as a function of the apparent wind angle. Naturally, the performance of the lake racer depends strongly on the sail usage, therefore, in order to define different sail setups, FS-Equilibrium introduces so called configurations. Following this approach, it is necessary to define four configurations that mirror the combined usage of all sails. Each configuration does not depend directly just on one force module, but can be used in any of them for the optimization process. When searching for the optimum trim condition the software cycles through all activated configurations, using the defined sails polars and picking the most promising configuration.

When sailing the high-performance lake racer, the parameter *Flat* plays a significant role for the transverse stability. The equilibrium for a displacing sailyacht is generated by the buoyancy when heeled. A VPP operates locating the crew in a specific position and heeling the yacht in order to find the equilibrium. The VPP for the lake racer has to follow a different approach as the yacht is handled differently. The heel angle is held constant by positioning the crew members and trimming the sails in order to achieve it. This handling is pictured in figure 6. The total righting moment is hence generated by the crew members hiking on the trapeze while flattening or de-flattening the sails. In the VPP, this behaviour is displayed by defining a control variable *MxCtrl*, which is linked to the parameters *Lat-crew* and *Flat*. If the heeling moment cannot be balanced by all four crew members hiking on the trapeze, the sails are de-powered by decreasing the *Flat* parameter. When the crew members have not yet reached the maximum hiking position, the sails are completely powered.



Figure 6: Crew of the Lake Racer Hiking [source: www.seahorsemagazine.com/165-content/1006-he-s-back (15.10.2021)]

The state value *Heel* describes the heeling angle of the lake racer in the VPP. According to its design requirements, the angle is expected to vary between +/- 10 degrees. Depending on the environmental condition and on how the yacht is handled in reality, this parameter can be held fixed or optimized, as presented further in the result discussion.

Naturally, the total parasitic resistance of the lake racer includes also the parasitic aerodynamic force caused by each object that is exposed in the wind above the waterline. For hydrofoiling sailing yachts, this component plays an especially significant role, as the achievable boat speeds are higher. By using the windage force module, the resistance is computed from the drag coefficients defined in three planes and from the respective projected areas of hull, crew, boom and bowsprit. The areas and respective centers are measured in provided drawings. The total drag coefficient is taken according to literature on aerodynamic drag (Hoerner, 1951). The standing rigging features, for example the spreaders, stays, jumpers, lines runners and wiring, are not considered due to their small influence. For the parasitic aerodynamic drag generated by the mast, another force module is introduced in the VPP model. The latter computes the mast resistance following a similar calculation using projected area, position, apparent wind deflection angle and drag coefficient.

PERFORMANCE ASSESSMENT

The setup of the VPP for the lake racer is then checked by means of monitoring the behaviour of settings, force modules and defined parameters. The forces acting on the lake racer in the VPP have to be implemented with accuracy considering the respective modelling criteria and assumptions. The role played by the different parameters has to describe in a realistic way the behavior of the sailing yacht. For this reason, while preparing the VPP to be run, some global plausibility checks are arranged. These monitor the output of every defined force module and variables. The monitoring process is conducted without optimizing any trim parameter, the program is only required to find steady state solutions.

Validation

Before running the VPP for the hydrofoiling lake racer in FS-Equilibrium, it is important to evaluate and quantify the accuracy of the model with the archimedean configuration, as the hydrofoil setup builds upon the archimedean setup. For this reason, during the validation process, carried out in the thesis (Melis, 2020), a comparison between the boat speeds realized in FS-Equilibrium and in an initial base VPP provided by the designer, is documented. The software which is used for the base VPP is not known and neither are the methods, assumptions and theories behind the models for the aerodynamic and

hydrodynamic forces. Therefore comparing the results of the two different VPPs is a speculative method of verifying validity here. The comparison between the speed polars reveals that the two VPPs compute, for the same TWSs and TWAs, different forces and hence different boat speeds. Both predictions show a strong dependency on TWA. FS-Equilibrium predicts lower boat speeds for small TWSs and larger boat speeds for greater TWSs. In between, the speed polars seem to follow the same trend and scales. Possible explanations for these divergencies are the usage of different aerodynamic models for the sail types and different approaches for the total hydrodynamic bare hull resistance. The VPP in FS-Equilibrium models the code0s as asymmetrical spinnakers, which can under- or overestimate their real efficiency depending on the apparent wind angle. Spinnakers generate greater aerodynamic side forces, so greater leeway angles, decreasing the boat speed. In stronger breezes this modelling error has smaller impact since the leeway angles decrease. The hydrodynamic bare hull resistance is modelled in FS-Equilibrium only by scaling the viscous resistance. Certainly, more precise explanations for the encountered divergences, could be made by comparing the VPP results with measured performance of the lake racer sailing in reality or with CFD calculations. In conclusion, despite of the established divergencies, the speed polars for the lake racer in the archimedean sailing mode are considered plausible.

Considerations

After having monitored the functionality of the archimedean VPP model for the lake racer, the hydrofoiling configuration can be activated in the software. By having a conservative and accurate design of the surface piercing hydrofoil, the software is rapidly able to find steady state equilibrium conditions. The appendage ensures flying stability during first investigations. For the trim parameter *Rake*, simulations for different values are conducted. The highest boat speeds are observed for the highest rake angle of 6 degrees across the range of TWA and TWS, where the hydrofoil generates the highest upwards lift. This is not surprising for a self-levelling surface piercing configuration of this type. More lift results in a higher ride height and less wetted surface area. It needs to be remembered that the appendage is retracted when not hydrofoiling, otherwise smaller *Rake* angles would be favorable in the hull borne regime. Fixing this trim parameter has some advantages for finding hydrofoiling equilibrium states and less required time of running the iteration. In order to monitor the broaching behaviour of the fully submerged elevator hydrofoil, by mean of rising out of the free waterline surface, a derived value is defined as *Elevator Immersion*. The parameter measures the distance between the free water surface and the sinkage of the elevator hydrofoil. In real sailing conditions the crew could prevent broaching and crashing by moving along the longitudinal axis of the yacht. Furthermore, the ability of the VPP to increase the performance of the lake racer model using the optimization algorithm, is investigated. The optimized trim parameters are the *Long-crew* position and the *Elevator* angle. These influence the pitch angle of the yacht, and hence also the effective *AoA* of the hydrofoils, impacting on the boat speed. The heeling behaviour certainly influences the performance of the yacht as well. For this reason, a VPP run is arranged optimizing the state value *Heel*. In these trials, the VPP is run twice, once with fixed trim parameter and once with activated trim parameter. For these variables, the optimization algorithm is able to find optimum values. The results are presented further in the discussion showing that these parameters influence as well the minimum hydrofoiling speed.

Another aspect to be considered for the real handling of the lake racer is the usage or non-usage of the surface piercing hydrofoil in the water. The crew is expected to drop and place the hydrofoil in defined positions with opportune wind strengths preparing to hydrofoil. Depending on the boat speed in which the hydrofoil is dropped in the water, the rake angles are to be adjusted smoothly without exceeding in lift too soon. Modelling the immersion of the hydrofoil, is arranged by the usage of new configurations coupled with the pre-existing ones for the sails. The program, while running, is able to pick for each condition whether the lake racer performs better with the surface piercing hydrofoil immersed in the water or not. In addition, considering an hydrofoiling sailboat, attention has to be drawn on the two equilibrium states conditioned by the resistance hump. These two balanced states are positioned before the hump, in hull borne condition and after the hump, in the foil borne condition (Abdel-Maksoud, 2020). By iterating from light into stronger winds, the VPP chooses the first steady state conditions which it finds, meaning those where the lake racer is still in hull borne condition. On the other hand, by iterating from strong into lighter winds, the VPP finds firstly the equilibrium in foil borne condition and a higher speed is computed. To facilitate the process of generating truthful speed polars for the hydrofoiling lake racer, an adjusted routine code, called in FS-equilibrium a macro expression, is defined and utilized to launch the simulations.

Results Discussion

Running the VPP of the lake racer with its hydrofoiling configuration delivers the displayed series of speed polars for different coloured TWSs. The tangential axis reports the TWAs and the radial axis shows the boat speed V_s . The speed polars displayed in figure 7 incorporate the hull borne and foil borne conditions. In this first configuration, the hydrofoils are set for the hydrofoil sailing mode with the *Elevator* and *Rake* fixed to constant values of 2 and 6 degrees respectively. The

position of the crew members is in the longitudinal direction, with the trim parameter *Long-crew*, also held equal to 0. The lake racer is sailing upright, by means of *Heel* held constant at 0 degrees. These first generated speed polars are used as a comparison for the following VPP runs aimed at optimizing the defined trim parameters. The light wind polars of 2 and 4 m/s TWS suggest that the archimedean sailing mode, in which the surface piercing hydrofoil is lifted out of the water, is more appropriate. In the speed polars, the acceleration indicator for the lake racer is clearly displaying an improved performance in foil borne condition. Only with stronger TWSs between 6 and 10 m/s the lake racer is able to rise out of the water in close reach courses. Depending on the TWS, it attains the hydrofoiling status until deep reaching courses. Further investigation regarding which minimum wind speed allows the lake racer to fully hydrofoil is carried out in the speed polars later on. For each stronger TWS in which the lake racer reaches the foil borne condition, the best VMGs for upwind and downwind are found, as expected, in broad reaching courses. The speed outcomes for tight close hauled courses are not considered relevant as they are unrealistic based on sailing physics. In reality, in order to reach the foil borne condition sooner, the yacht is firstly sailed on a reaching course and then closed hauled. The transition between archimedean and hydrofoil sailing mode in stronger breezes of 10 m/s, happens, naturally, at smaller TWAs compared to the polar for TWS 6 m/s. This is because the lake racer reaches the minimum hydrofoiling speed at an earlier stage. On the other hand, the transition from hull to foil borne in deep running courses for TWS of 10 m/s is delayed when compared to TWS of 6 m/s. The yacht is still capable of sailing above the minimum hydrofoiling speed. In these courses, the speed performance of the lake racer drops radically. The yacht finds its equilibrium in the hull borne condition and the apparent wind speed drops dramatically. It must be mentioned that these courses are rarely sailed, since the VMG is found at reaching angles. A tactician would rather choose the longer distance reaching course with greater VMG, rather than the running course, in which the yacht is very often unstable.

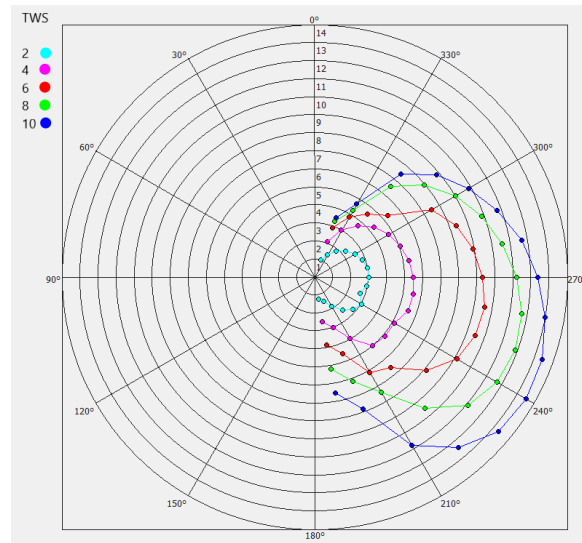


Figure 7: Unoptimized Performance Assessment

Another factor that can influence the lake racer's performance in archimedean and hydrofoil sailing mode, is the heel angle. This investigation shows the performance variation by optimizing the state value *Heel* in comparison to the upright condition. Figure 8a and 8b provide the resulting speed polars and the computed optimum heel angles. The angle is positive when heeling to windward and negative when heeling to leeward. The optimization results of tight close hauled and deep running courses are not included in this analysis as they are not realistic for any TWS. The plots displayed in figure 8b present a few large heel changes and kinks for specific TWAs, which can be explained by changes of sail configuration. For TWS of 2 m/s, the optimization algorithm computes positive upwind heel angles and negative downwind angles. This inquirable behaviour of the VPP does not mirror the real handling of the lake racer. A plausible reason for this is the rudder elevator which generates an additional side force beside the upwards lift. When the yacht heels to windward this side force acts to windward and is beneficial. However, the effect is small and the performance gain in TWS of 2 m/s is barely visible as the wetted surface area remains almost constant. For TWS of 4 m/s, the lake racer heels leeward over all reaching courses. In this condition the crew is positioned in the most windward hiking position. The leeward heel increases the righting moment and the sails can be powered up further. The boat speed increases slightly compared to the upright sailing condition. As expected, considering the flat hull bottom and the positions of buoyancy and gravity centers, the boat speed does not increase significantly when heeling in the archimedean sailing mode. On the contrary, the performance increase when hydrofoiling with optimized heel angle, can be noticed in figure 8a. In most hydrofoiling conditions, the optimum heel angle is to leeward as well. According to the output

tables of the VPP, the reaching leeway angles are smaller compared to the upright sailing condition. When heeled to leeward, the surface piercing hydrofoil generates less leeward side force and the induced drag decreases. In addition it is found that the pitch angle is coupled with the heeling behavior. When the yacht heels, it rotates around the lift center of the hydrofoils. This behavior depends on the hydrofoil's positions and on the gravity centers. With unoptimized heel angle, the upright lake racer is sailing slightly bow-down because of the fixed parameters *Long-crew* and *Elevator*. In the heel optimization the bow-down pitch angle is reduced, increasing the effective *AoA* of the hydrofoils. Compared to the performance with fixed heel the ride height is greater, decreasing the wetted surface area of the appendages while increasing the boat speed. As shown by the plots in figure 8b, the optimum leeward heel angle decreases with increasing TWS. In stronger breeze conditions, when hydrofoiling upwind, the optimum heel angles are larger than on downwind courses. Optimum heel angles of approximately 0 degrees are computed for broad reaching courses, where the highest downwind VMG is found. For TWS of 10 m/s, the VPP computes small windward heel angles on broad reaching courses. In these conditions the ride height is quite significant and the leverage arm of the hiking crew is increased by the windward heel. According to the results, for TWA of 120 degrees, with a windward heel angle of 0.8 degree, the boat speed increases by approximately 1 knot.

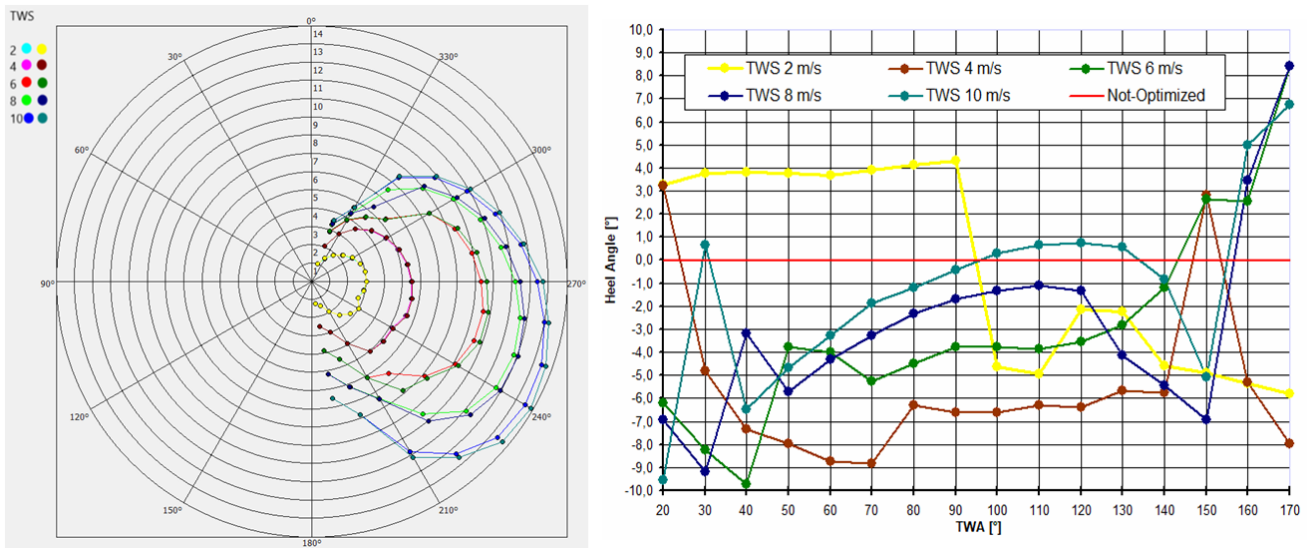


Figure 8: Optimized Performance Assessment (a) and Optimized *Heel* as Function of TWA for each TWS (b)

Further analysis is carried out by optimizing the position of the five crew members by activating *Long-crew* as a trim parameter. It varies between 1, which indicates the crew being positioned in the most forward part, and -1 when the crew is placed in the most rear part of the cockpit. Figure 9a displays two VPP runs. In the first one, the *Long-crew* is held constant at 0, while in the second the trim parameter is optimized improving slightly boat speed. Generally, for regatta yachts like the lake racer, shifting the crew positions forward while sailing upwind, or to stern while sailing downwind, increases the speed of the yacht. Figure 9b reports the crew positions as function of TWA for each TWS in all steady state conditions of the optimized and un-optimized run. By viewing the plots for this trim parameter in strong winds, the crew placement in the foreship sailing upwind and in the aftship when sailing downwind becomes visible. For light winds and very small or very large TWAs in stronger winds, the lake racer is sailing in the archimedean mode reaching a maximum boat speed of approximately 6 m/s. In such conditions, the optimized run of the program does not deliver visible changes of performance. When running the optimization with this boat speed regime, the algorithm places the crew members as much as possible in the foreship, as evidenced by *Long-crew* = 1. The reason for this is that, in the foreship, the wetted surface area is slightly reduced compared to the aftship. Thus, the VPP tries to minimize it by increasing the bow-down pitch. For lighter TWSs, this behaviour of the VPP model can be considered realistic. According to the speed polars, the optimization of the longitudinal crew position, also impacts the transition. The upwind transition, for TWSs of 8 and 10 m/s, takes place slightly delayed compared to the run for the un-optimized trim parameter. One possible reason for this divergence, is the optimization algorithm, which focuses on local equilibrium condition. During the optimization the algorithm may therefore not find the other equilibrium condition due to fairly limited state and force variation. When optimizing the parameter for broad reach courses, the yacht transits back to hull borne state under greater boat speeds, compared to the unoptimized run. By comparing the displayed VPP runs and the output tables in the software, it can be noticed that while hydrofoiling with fixed *Long-crew* parameter, the yacht sails with greater bow-down pitch angles. As displayed by the speed polars, the lake racer is sailing in hydrofoil mode in reaching courses with TWS of 6,8 and 10 m/s. In these conditions, according to figure 9b, the optimization algorithm shifts

the crew members to the center or to the rear of the cockpit by means of *Long-crew* between 0 and -1. The bow-down pitch is reduced, which results in a higher effective AoA of the hydrofoils. Thus, the ride height is greater and the total hydrodynamic resistance is reduced. As the boat speed increases, its improvement becomes visible for stronger TWSs in downwind courses.

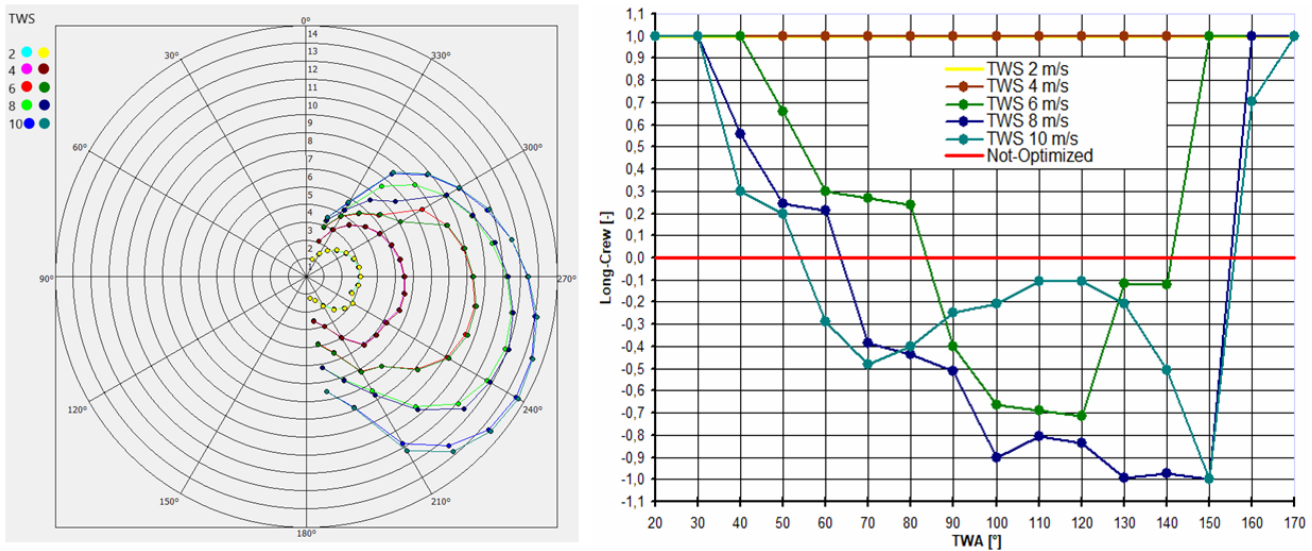


Figure 9: Optimized Performance Assessment (a) and Optimized *Long-crew* as Function of TWA for each TWS (b)

Additionally, in the following run of the VPP model, the trim parameter *Elevator* is activated. The unoptimized and optimized runs are conducted by keeping the crew positioned in the middle of the cockpit, meaning that the lake racer is sailing with a slight bow-down pitch. Figure 10a displays two VPP runs. In the first one the *Elevator* is held constant at 2 degrees while in the second, the trim parameter is optimized. Figure 10b reports the elevator angle as function TWA for each TWS in all steady state conditions of the optimized and un-optimized run. The optimization results of tight close hauled and deep running courses are omitted in this analysis since it would not be realistic for any TWSs. In light winds the lake racer is sailing in its archimedean configuration with the surface piercing hydrofoil lifted out of the water. According to figure 10b, in conditions with TWS 2 m/s, the optimization algorithm provides an *Elevator* angle between 0 and 1 degrees over different TWA. As shown, for TWS of 4 m/s, the optimized elevator angle reaches a maximum value of 5 degrees.

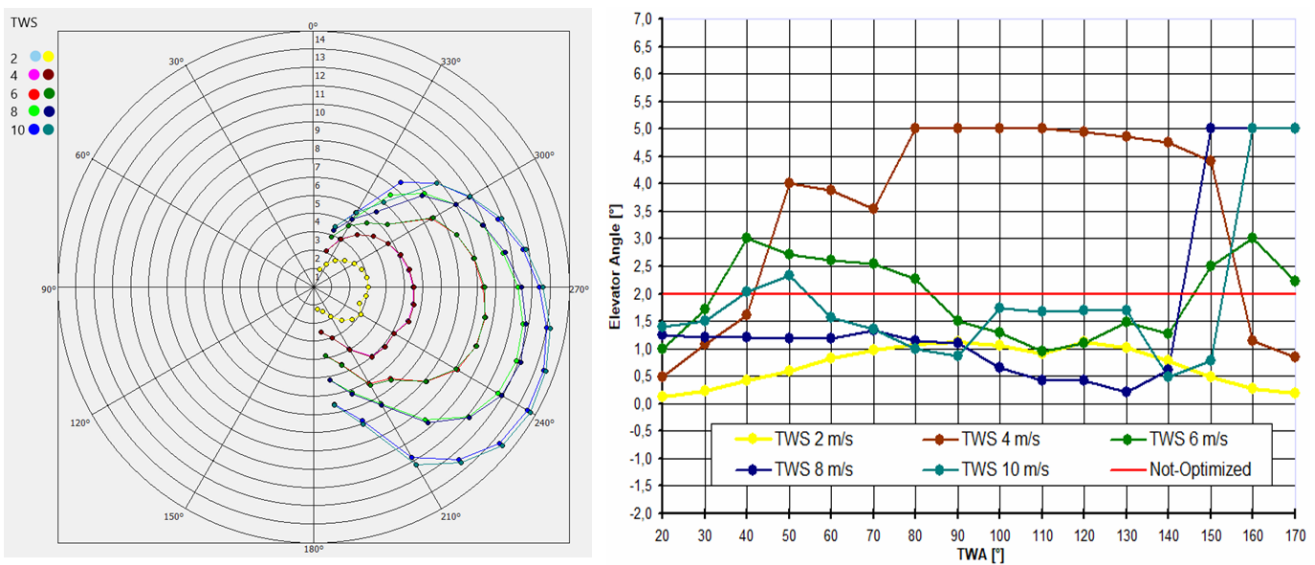


Figure 10: Optimized Performance Assessment (a) and Optimized *Elevator* as Function of TWA for each TWS (b)

The algorithm is trying to reduce the wetted surface area of the aftship in the archimedean sailing mode by increasing the elevator angle and the *Lift Fraction*. Figure 10a shows, however, that the performance gain is barely visible. In medium breezes of TWS 6 m/s, the lake racer is able to attain the hydrofoil sailing mode on reaching courses. In upwind conditions of TWS 6 m/s, the optimum *Elevator* angle appears to be between 1 and 3 degrees depending on TWA. While sailing downwind, instead, the optimum angle is found in the interval between 1 and 1.5 degrees. The optimized run of the program does not deliver visible performance changes for this condition. The major boat speed gains can be noticed with stronger TWSs of 8 and 10 m/s starting from beam reaching to tight running courses. In hydrofoiling conditions, the optimization algorithm computes lower elevator angles due to greater incoming flow speed on the appendage. According to the output table of the VPP, the bow-down pitch of the yacht is decreased, resulting in higher effective *AoA* of the surface piercing hydrofoil, increased ride height and consequently greater boat speed.

Another interesting comparison is delivered by figure 11: the performance assessment in archimedean and hydrofoil sailing mode. In the first configuration the surface piercing hydrofoil is completely lifted out of the water in all environmental conditions and the elevator rudder is fixed at zero angle for minimal drag. In the second one, the hydrofoil parameters *Rake* and *Elevator* are again fixed to constant values of 6 and 2 degrees respectively. The algorithm chooses whether it is more efficient to sail with or without the immersed surface piercing hydrofoil. In both setups the longitudinal crew position is held constant and the lake racer is sailing upright. The diagram contains a total of 11 speed polars: five for the archimedean sailing condition and six for the hydrofoiling condition. In order to visualize under which environmental circumstances the lake racer transits from the archimedean to the hydrofoil sailing mode, the yellow speed polar is generated additionally for TWS of 5 m/s. Regarding the results in light winds, for example TWS of 2 and 4 m/s, the yacht performance does not change significantly between the two sailing configurations according to the VPP. For TWS 2 m/s, the small resistance change, generated by the different elevator angles, is barely visible.

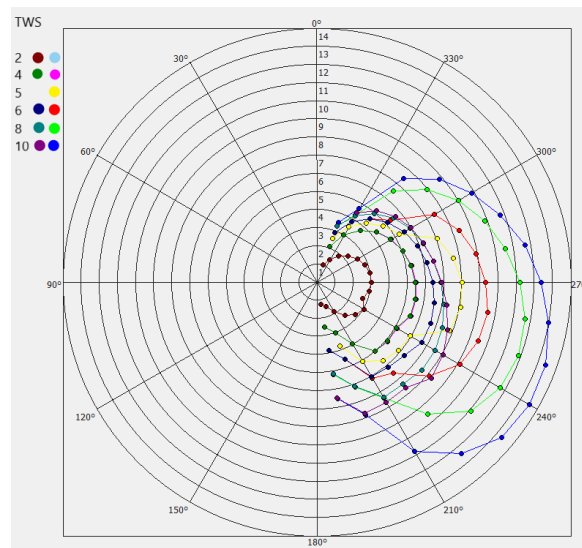


Figure 11: Performance Increase between Archimedean and Hydrofoil Sailing Modes

In environmental conditions with TWS 4 m/s, the lake racer is expected to sail in the archimedean mode. The VPP predicts a slightly higher boat speed when the hydrofoiling configuration of the elevator is fixed at 2 degrees. For this TWS, considering figure 10b, the elevator angle equal to 0 degrees is the less efficient while, 5 degrees is the most efficient elevator angle. Increasing the elevator angle results in a higher bow-down pitch, the *Lift Fraction* increases slightly reducing the wetted surface area of the bare hull. Furthermore, it is found that for a TWS of 5 m/s, the lake racer is already able to attain a fully hydrofoiling state at beam reaching angles. Obviously, by comparing the performance assessments for both configurations in stronger TWSs of 6 to 10 m/s, the speed gain is significant and increases with the wind strength. The sooner the yacht is lifted out of the water, the more rapidly the boat speed increases. The induced drag of the appendages decreases with the squared boat speed. The ride height increases and the wetted surface area is thus also reduced. The lake racer perceives a strong acceleration and this concept summarizes the hydrofoil lift-drag "trade-off" of high-performance flying sail yachts. For example, in an environmental condition of 90 degrees TWA and TWS of 10 m/s, the archimedean boat speed of 6.8 m/s, is increased to 12.5 m/s with the hydrofoiling configuration.

The transition between archimedean and hydrofoiling modes is often quite complex to model in a VPP, since the algorithm bounces between steady state conditions in different environments. However, most likely due to the self-stabilizing hydrofoils configuration, the simulation of the transition regime has turned out successfully. In table 2 a detailed case study for the transition mode is conducted for TWS of 5,6,8 and 10 m/s. Defining the minimum hydrofoiling speed as the boat speed which is required to attain a fully hydrofoil sailing condition, it must be kept in mind that the table does not provide feedback regarding the take-off speed. In real sailing situations the take-off speed can differ from the minimum hydrofoiling speed. For example when encountering a resistance hump that needs to be overcome, the take-off speed is higher than the minimum hydrofoiling speed. Alternatively, dynamic effects such as crew movement can cause the take-off speed to be lower than the minimum hydrofoiling speed. Because these effects are not considered in this VPP, no conclusions regarding the take-off speed can be drawn. In table 2, the TWA is gradually changed, starting from higher to lower TWAs, so that the balance algorithm iterates starting from a hydrofoil sailing condition. In this way, the minimum upwind hydrofoiling speed is obtained from the lowest TWA where the *Lift Fraction* is still 1. Iterating the other way around would mean that the balance algorithm starts from an archimedean sailing condition and may find the archimedean equilibrium condition instead of the hydrofoiling equilibrium if two equilibrium states exist. During this first investigation, trim and state parameters such as *Rake*, *Heel*, *Elevator* and *Long-crew* are not adjusted.

Table 2: Steady State Conditions Transition Mode Upwind for each TWS

C	TWS	TWA	TWC	AWA	Vs	VMG	Leeway	Rake	Pitch	Elevator	ElevatorImmersion	Sink	LiftFraction	WSA	Configs
●	5.00	63.00	65.92	25.54	<u>6.41</u>	2.61	2.92	6.00	-2.38	2.00	1.29	-0.291	1.00	0.00	JibA
●	5.00	62.99	66.28	28.30	5.44	2.19	3.29	6.00	-0.25	2.00	1.43	-0.075	0.59	7.02	JibA
●	5.00	55.00	58.51	25.89	4.96	2.59	3.51	6.00	-0.20	2.00	1.45	-0.058	0.49	7.71	JibA
●	6.00	51.90	54.71	23.20	<u>6.60</u>	3.81	2.81	6.00	-2.13	2.00	1.24	-0.328	1.00	0.00	JibA
●	6.00	51.89	55.21	25.71	<u>5.46</u>	3.12	3.32	6.00	-0.30	2.00	1.43	-0.078	0.60	6.96	JibA
●	6.00	48.00	51.47	24.42	5.14	3.20	3.47	6.00	-0.26	2.00	1.44	-0.065	0.53	7.42	JibA
●	8.00	38.90	41.89	20.54	<u>6.34</u>	4.72	2.99	6.00	-2.69	2.00	1.28	-0.314	1.00	0.00	JibA
●	8.00	38.89	42.37	22.38	<u>5.21</u>	3.85	3.48	6.00	-0.31	2.00	1.44	-0.070	0.55	7.29	JibA
●	8.00	38.50	42.00	22.22	5.17	3.84	3.50	6.00	-0.31	2.00	1.44	-0.068	0.54	7.35	JibA
●	10.00	33.00	36.07	19.30	<u>6.26</u>	5.06	3.07	6.00	-2.93	2.00	1.28	-0.318	1.00	0.02	JibA
●	10.00	32.99	36.58	20.89	<u>5.06</u>	4.07	3.59	6.00	-0.31	2.00	1.45	-0.065	0.52	7.53	JibA
●	10.00	31.00	34.72	19.91	4.80	3.95	3.72	6.00	-0.29	2.00	1.45	-0.055	0.47	7.91	JibA

By analysing the underlined boat speeds, it is possible to draw some conclusions regarding the minimum upwind hydrofoiling speeds of the modelled lake racer. In addition, comparing these with the minimum hydrofoiling speed provided by NC Raceboats, the plausibility of the VPP can be evaluated. For an initial concept design of the surface piercing appendage a minimum hydrofoiling speed of 6 m/s was predicted. According to table 2, the VPP predicts minimum upwind hydrofoiling speeds of 6.41, 6.60, 6.34 and 6.26 m/s for TWS of 5,6,8 and 10 m/s respectively. The minimum hydrofoiling speed is driven by the effective *AoA* of the surface piercing hydrofoil. Depending on the resulting pitch angle when reaching, a minimum hydrofoiling speed around 6 m/s as intended seems realistic. During a further investigation illustrated in table 3 for TWA of 90 degrees, the minimum hydrofoiling speed is determined by reducing the TWS until the upright lake racer just stays on its hydrofoils. In the displayed conditions, different *Elevator* angles and different *Long-crew* positions are used to realise increasing bow-up pitch (negative sign) and hence increasing the effective *AoA* of the hydrofoils. The resulting minimum hydrofoiling speed and associated minimum TWS is shown for each *Elevator* and *Long-crew* combination. It can be seen that the reaching minimum hydrofoiling speed reduces as the bow-up pitch increases; down to 5.99 m/s. The required minimum TWS increases slightly with reducing boat speed since the induced drag increases.

Table 3: Analysis of Reaching Minimum Hydrofoiling Speed

C	TWS	TWA	TWC	AWA	Vs	VMG	Leeway	Rake	Pitch	Elevator	ElevatorImmersion	Sink	LiftFraction	WSA	Long_crew
1	4.295	90.0	92.78	31.53	6.52	-0.32	2.78	6.00	-2.03	2.00	1.30	-0.271	1.00	0.00	0.00
2	4.305	90.0	92.67	33.44	6.13	-0.29	2.67	6.00	-3.38	3.00	1.23	-0.382	1.00	0.00	-0.50
3	4.410	90.0	92.59	34.88	<u>5.99</u>	-0.27	2.59	6.00	-4.49	4.00	1.19	-0.459	1.00	0.00	-1.00

CONCLUSIONS

With help of the software tool FS-Equilibrium, a quasi-static velocity prediction program for the hydrofoiling lake racer was developed based on first principal semi-empirical assumptions. Different configurations of the software were considered, which led then to the appropriate selection of the force modules based on their underlying theories. In order to mirror the practical handling of the lake racer, different parameters were defined and linked to the respective force modules. After having monitored the correct functionality of the installed setup carrying out plausibility checks, the VPP was utilized. As expected,

the performance of the yacht in hydrofoiling condition is significantly greater compared to the archimedean condition. In light breezes, the yacht sails on its archimedean configuration with the un-submerged hydrofoil appendage. In medium breezes, instead, the lake racer is already able to stand up on its hydrofoils and attain a stable flight mode. Several aspects of the real lake racer have been simplified. The sail forces are simulated using a semi-empirical approach. The total hydrodynamic resistance is modelled by using a scaled form factor and the actual wetted surface area. Force components such as the spray resistance and the added resistance (not a major factor when hydrofoiling on lakes) are also not considered. Similarly, effects such as cavitation and ventilation are also disregarded. Despite of the limitations, the VPP produces plausible results, which appear applicable for initial performance assessment and design studies. Future studies on the VPP should therefore be validated with real-life measurements and observations. The prediction and assessments contained in this paper demonstrate that it is possible to develop a stable VPP model for a hydrofoiling sailing yacht using simple assumptions and theories.

AUTHOR BIOGRAPHY

Michele Francesco Melis grew up bilingual, German and Italian on an island called Sardinia, Italy. After finishing high school between Cagliari, Italy and Saint Augustine, Florida, he started his naval architecture bachelor of science at the Hamburg University of Technology (TUHH) in which he graduated with this project. Currently he studies in the ship science programs between the Hamburg University of Technology and the University of Southampton, focusing on hydro-aerodynamics, performance prediction and wind propulsion of sailing yachts.

ACKNOWLEDGEMENTS

The author would like to thank his external advisers Dr. Heikki, Dr. Karsten Hochkirch from the Ship Performance Center of DNV and Dr. Martin Fischer from NC Raceboats. Furthermore, the author wants to thank Ing. Thomas Jundt for allowing him to model his hydrofoiling Lake Racer and his faculty advisers Professor Abdel-Maksoud and Dr. Roland Gosda.

REFERENCES

- ABBOTT I. H., *Theory of Wing Sections: Summary of Airfoil Data*, New York - NASA, 1959
- ABDEL-MAKSoud M., *Hydrodynamics of High Speed Vehicles*, Institute for Fluid Dynamics and Ship Theory - TUH, 2020
- DNV, *FS-Equilibrium User Manual*, Hamburg, 2021
- FALTINSEN O. M., *Hydrodynamics of High-Speed Marine Vehicles*, Cambridge University Press, 2009
- HANSEN H., HOCHKIRCH K., BURNS I., FERGUSON S., *Maneuver Simulation and Optimization for AC50 Class*, Journal of Sailing Technology - The Society of Naval Architects and Marine Engineers, 2019
- HOCHKIRCH K., *PhD Thesis: Design and Construction of a Full Scale Measurement System for the Analysis of Sailing Performance*, Mensch und Buch Verlag - Technische Universität Berlin, 2000
- HOOKE R., JEEVES T. A., *Direct Search Solution of Numerical and Statistical Problems*, Journal of the ACM Vol 8, 1961
- ITTC 1957, *International Towing Tank Conference 1957*, Madrid Spain, 1959
- KEUNING J., KATGERT M., *A Bare Hull Resistance Prediction Method Derived from the Results of the DSYHS Extended to Higher Speeds*, TU-Delft, 2008
- MELIS M. F., *Bachelor Thesis: Velocity Prediction Program for the Hydrofoiling Lake Racer QFX with the Software Tool FS-Equilibrium*, Institute for Fluid Dynamics and Ship Theory- TUH, 2020
- ORC, *International Measurement System - Offshore Racing Congress Racing System*, 2001
- PAULIN A., HANSEN H., HOCHKIRCH K., FISCHER M., *Performance Assessment and Optimization of a C-Class Catamaran Hydrofoil Configuration*, 5th High Performance Yacht Design Conference Auckland, 8-12 March 2015
- PRANDTL L., *Applications of Modern Hydrodynamics to Aeronautics (NASA)*, Göttingen, Germany 1923
- WHICKER L. F., FEHLNER L., *Free Stream Characteristics of a Family of Low Aspect ratio All Moveable Control Surfaces for Application to Ship Design*, DTMB Report 993, 1958

[This paper has been presented at the 24th Chesapeake Sailing Yacht Symposium on the 11.06.2022 in Annapolis, USA]